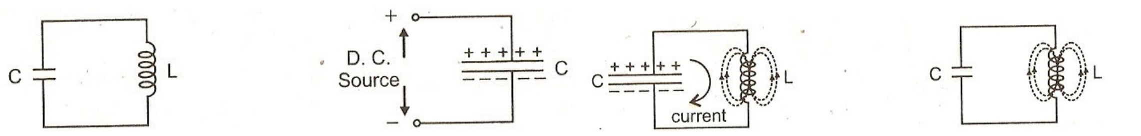
L - 8: Analysis of LC oscillators,

LC Oscillators are oscillators which use elements $L and C$ to produce oscillations. The circuit using elements $L and C$ is known as *tank* or *oscillatory* or *resonant* or *tuned* circuit. These oscillators are used for high frequency range (200 kHz to GHz)

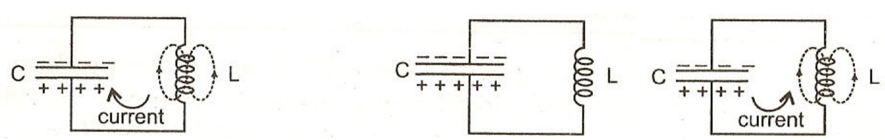
1. How a LC tank circuit works



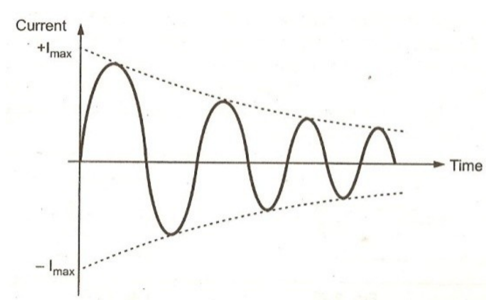
LC tank circuit   Initial charging   Discharging   Magnetic fields in L

When the capacitor C gets charged, electrostatic energy gets stored in the capacitor. When the charged capacitor is connected across an inductor L, it discharges through L. Due to such current flow, the magnetic gets set up around the inductor L and the inductor starts storing the energy. When the capacitor is fully discharged, maximum current flows through the circuit, and at this instant all the electrostatic energy gets stored as magnetic energy in the inductor.

When the magnetic field around L starts collapsing, the capacitor is charged with opposite polarity (as per Lenz's law). After some time the capacitor gets fully charged with opposite polarities meaning that the entire magnetic energy has been converted back to electrostatic energy in capacitor, since the process in continuous an alternating (oscillatory) current is produced in the tank circuit.



The energy transfer from C to L and L to C losses occur due to which the amplitude of oscillating current decays exponentially. Damped oscillations stop after some time.



To make oscillations sustain, an amplifier can be used to compensate the energy which is lost.

The frequency of oscillations generated by a tank circuit depends on $L \, and \, C$ values.

$$f = \frac{1}{2\pi\sqrt{LC}} Hz \quad Where; \quad L - henries; \; C - farads$$

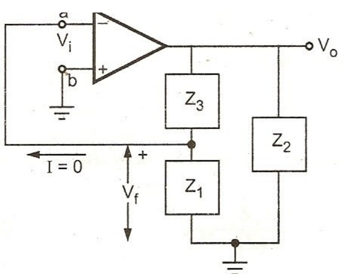
Depending upon the type of tank circuits, LC oscillators are classified as

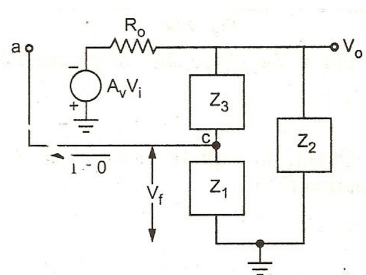
- *Colpitt's Oscillators;*
- *Hartley Oscillators;*
- *Clap Oscillators*;

2. Basic forms of LC circuit

The circuit consists of an amplifier stage of gain A_V with its output to a feedback stage of impedances $Z_1, Z_2 \, and \, Z_3$. The amplifier provides a phase shift of 180^0 while the feedback network provides an additional phase shift of 180^0

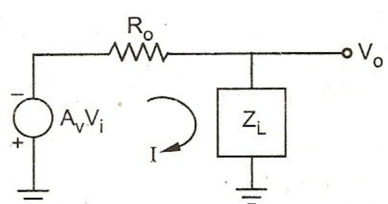
Basic Circuit





Equivalent Circuit

Analysis of the amplifier stage;



The input impedance of the amplifier is infinite; hence there is no current flowing towards the input terminal i.e. $I=0$. Let R_0 be the output impedance of the amplifier stage.

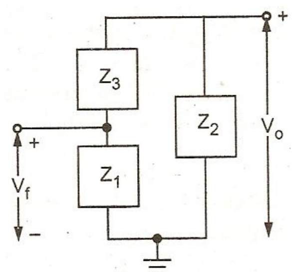
As $I=0$, Z_1, Z_2 appear in series, $Z_1, Z_2 \, and \, Z_3$ form an equivalent load impedance Z_L .

Where $Z_L = \frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}$

$$\textit{Therefore}\ \ \mathrm{I} = \frac{-\mathrm{A}_V V_i}{R_0+Z_L} \quad \textit{Since}\ \ V_0 = IZ_L \Rightarrow \ V_0 = \frac{-\mathrm{A}_V V_i Z_L}{R_0+Z_L}; \quad \textit{Hence}\ \ \frac{V_0}{V_i} = A = \frac{-\mathrm{A}_V Z_L}{R_0+Z_L}$$

Where A , is the gain of the amplifier stage.

<u>Analysis of the feedback stage;</u>



$V_f = V_0\left[\frac{Z_1}{Z_1+Z_3}\right], \textit{Therefore}\ \frac{V_f}{V_0} = \beta = \left[\frac{Z_1}{Z_1+Z_3}\right],$

But as the phase shift of the feedback network is 180^0 ,

then $\beta = -\left[\frac{Z_1}{Z_1+Z_3}\right]$.

To satisfy Barkhausen Criterion $-A\beta = 1; \quad \textit{Therefore}\ \ -A\beta = \frac{-A_V Z_1 Z_L}{(R_0+Z_L)(Z_1+Z_3)}$

Hence
$$-A\beta = \frac{-A_V Z_1\left[\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right]}{\left(R_0+\left[\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right]\right)(Z_1+Z_3)}$$

Dividing numerator and denominator by $\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}$ and substituting

$$Z_1 = jX_1, Z_2 = jX_2 = and\ Z_3 = jX_3\ Where\ X = \omega L - \ \text{inductance}\ \mathrm{X} = \frac{-1}{\omega C} - \text{capacitance}$$

$$-A\beta = \frac{-A_V X_1 X_2}{-X_2(X_1+X_3) + jR_0(X_1+X_2+X_3)}$$

To have a phase shift of 180^0 , the imaginary part of the denominator must be equal to zero i.e. $X_1+X_2+X_3 = 0 \ \ \Rightarrow X_1+X_3 = -X_2$

Hence
$$-A\beta = \frac{-A_V X_1 X_2}{-X_2(X_1+X_3)}\ or -A\beta = \frac{-A_V X_1}{-X_2} = A_V\frac{X_1}{X_2}$$

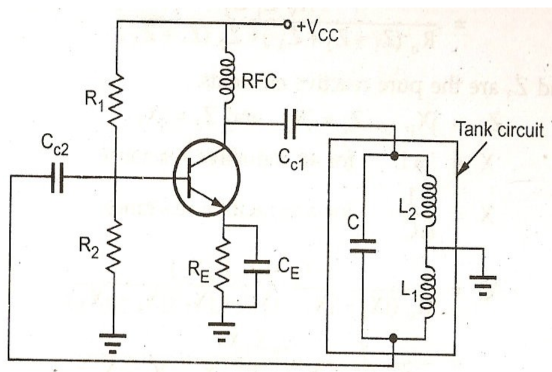
According to Barkhausen Criterion $-A\beta$ must be positive and greater than unity. As A_V is positive $-A\beta$ will be positive only if $X_1\, and\, X_2$ will have the same sign. **This indicates that $X_1\, and\, X_2$ must be both inductive or both capacitive.**

Oscillator type	Reactance element in tank circuit		
	X_1	X_2	X_3
Hartley Oscillator	L	L	C
Colpitt's Oscillators	C	C	L

3. Analysis of Hartley Oscillators

- **Transistorized Hartley Oscillator**

The amplifier stage uses a transistor in common emitter configuration.



Functions of amplifier stage components

$R_1 \, and \, R_2$ - biasing resistance;

RFC (Radio frequency choke)- isolation between a.c. and d.c. due to its high reactance at high frequencies and zero reactance for d.c.

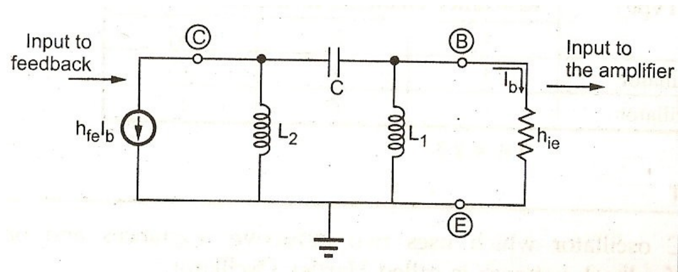
R_E -emitter circuit biasing resistor;

C_E -emitter bypass capacitor;

$C_1 \, and \, C_2$ -coupling capacitor

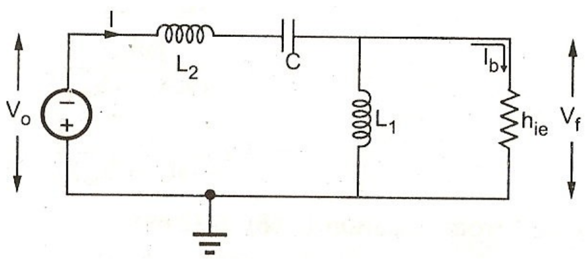
The common emitter amplifier provides a phase shift of 180^0. As the emitter is grounded, base and collector voltages are out of phase by 180^0. As the center of $L_1 \, and \, L_2$ is grounded, when the upper end becomes positive, the lower becomes negative and vice versa, so the LC feedback network gives an additional phase shift of 180^0 required to satisfy the necessary conditions for oscillation.

<u>*Derivation of Frequency of Oscillations*</u>



The output current is the collector current $I_C = h_{fe} I_b$ where I_b is the base current, h_{ie} the input impedance of the transistor. The output of the feedback is I_b , which is the input current of the transistor.

Using a simplified equivalent diagram after converting current source into voltage source;



$$V_0 = h_{fe} I_b X_{L2} = h_{fe} I_b \, j\omega L_2$$

$L_1 \, and \, h_{fe}$ are in parallel, then the current I drawn from the supply

$$I = \frac{-V_0}{\left[X_{L2} + X_C\right] + \left[X_{L1} \| h_{ie}\right]}$$

NB – The –ve sign indicates current direction shown in opposite to the polarities of V_o

Substituting $X_{L2} + X_C = j\omega L_2 + \frac{1}{j\omega C}$ and $X_{L1} \| h_{ie} = \frac{j\omega L_1 h_{ie}}{j\omega L_1 + h_{ie}}$ in the current equation

$$I = \frac{-h_{fe} I_b \, j\omega L_2}{\left[j\omega L_2 + \frac{1}{j\omega C}\right] + \left[\frac{j\omega L_1 h_{ie}}{j\omega L_1 + h_{ie}}\right]} \qquad \text{(i)}$$

According to current division in parallel circuits $I_b = I \times \frac{X_{L1}}{X_{L1} + h_{ie}} = I \times \frac{j\omega L_1}{j\omega L_1 + h_{ie}}$ (ii)

Substituting the value of I and rationalizing the denominator we get;

$$I = \frac{\omega^4 h_{fe} L_1^2 L_2 C \left(1 - \omega^2 L_2 C\right) + j\omega^3 h_{fe} L_1 L_2 C \left[h_{ie} - \omega^2 C h_{ie} \left(L_1 + L_2\right)\right]}{\left[h_{ie} - \omega^2 C h_{ie} \left(L_1 + L_2\right)\right]^2 + \omega^2 L_1^2 \left(1 - \omega^2 L_1^2 L_2 C\right)^2} \qquad \text{(iii)}$$

To satisfy the conditions, the imaginary part of RHS must be equal to zero;

$$Therefore\quad \omega^3 h_{fe} L_1 L_2 C\left[h_{ie} - \omega^2 C h_{ie}\left(L_1 + L_2\right)\right] = 0 \quad Hence \quad \omega = \frac{1}{\sqrt{C\left(L_1 + L_2\right)}}$$

$\Rightarrow f = \dfrac{1}{2\pi\sqrt{C\left(L_1 + L_2\right)}}$ - The frequency of oscillation. If

$L_1 + L_2 = L_{eq} \quad then\, f = \dfrac{1}{2\pi\sqrt{CL_{eq}}}$

Equating the value of $\omega = \dfrac{1}{\sqrt{C\left(L_1 + L_2\right)}}$ in eq, (iii) when the imaginary part is 0 the value of h_{fe} required to satisfy the oscillating conditions is $h_{fe} = \dfrac{L_1}{L_2}$. For mutual inductance of M $h_{fe} = \dfrac{L_1 + M}{L_2 + M}$

Example

In a transistorized Hartley oscillator two inductors are 2mH and 20μHwhile the frequency is to be varied from 950 kHz to 2050 kHz. Calculate the range over which the capacitor is to be varied.

Solution

The frequency is given by $f = \dfrac{1}{2\pi\sqrt{CL_{eq}}}\, where\, L_{eq} = L_1 + L_2$

$$L_{eq} = 2\times10^{-3} + 20\times10^{-6} = 0.00202$$

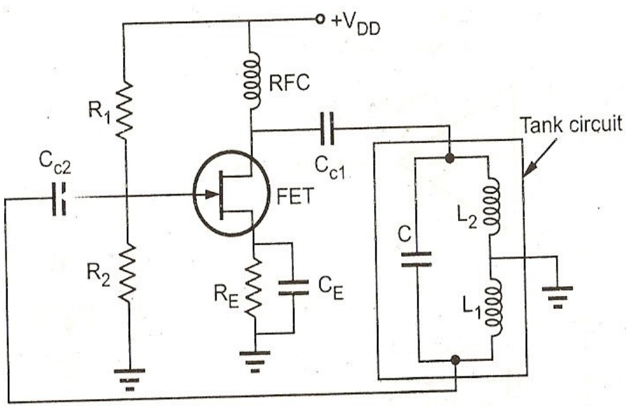
$$When\, f = f_{\max}\ ;\ 2050\times10^3 = \frac{1}{2\pi\sqrt{C\times0.00202}} \quad \Rightarrow \quad C = 2.98\, pF$$

$$When\, f = f_{\min}\ ;\ 950\times10^3 = \frac{1}{2\pi\sqrt{C\times0.00202}} \quad \Rightarrow \quad C = 13.89\, pF$$

- *FET* **Hartley Oscillator**

A *FET* is used as an active device in an amplifier stage. The resistors R_1, R_2 bias the FET along with R_3 to maintain Q point stable, coupling capacitors

C_{C1}, C_{C2} with larger values compared to C are used.



Refer;

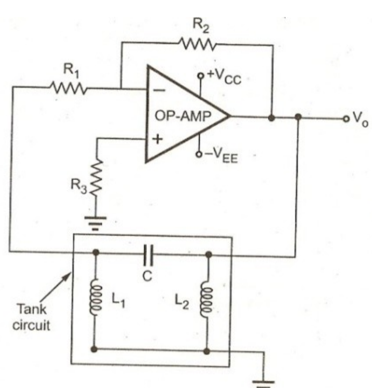
$$X_1 + X_2 + X_3 = 0 \text{ and } X_1 = j\omega L_1,\ X_2 = j\omega L_2 \text{ and } X_3 = \frac{1}{j\omega LC}$$

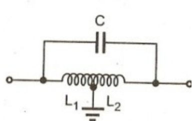
Solving for ω we get $f = \frac{1}{2\pi\sqrt{CL_{eq}}}$; where $L_{eq} = L_1 + L_2 \text{ or } L_1 + L_2 + 2M$

If $L_1 = L_2 = L$, then the frequency of oscillations becomes $f = \frac{1}{2\pi\sqrt{CL}}$

- **Hartley Oscillator using Op-amp**

The amplifying stage uses an op-amp as an active device





The frequency of oscillations

$$f = \frac{1}{2\pi\sqrt{CL_{eq}}} \quad \text{where} L_{eq} = L_1 + L_2 \text{ or } L_1 + L_2 + 2M$$

For oscillations, the amplifier gain $A_V = \frac{L_1}{L_2}$

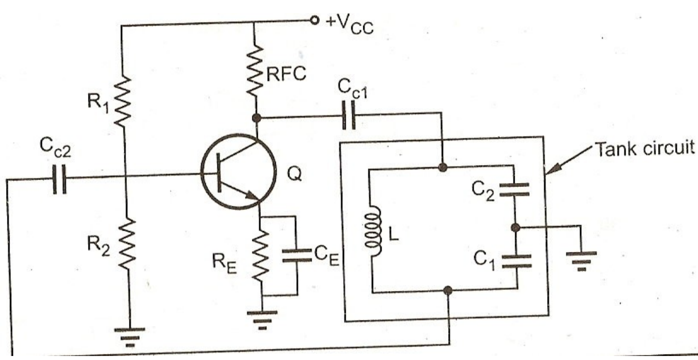
If mutual inductance exists between L_1 and L_2, then $A_V = \frac{L_1 + M}{L_2 + M}$

4. Analysis of Colpitt's Oscillator

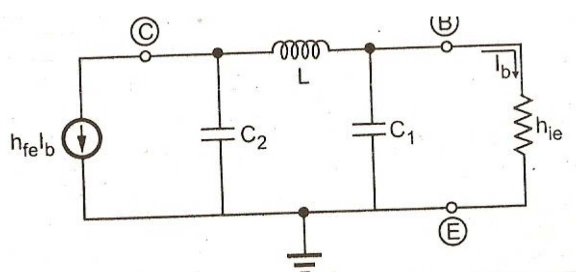
A Colpitt's oscillator is a high frequency LC oscillator (between 1 MHz – 500 MHz) which uses two capacitive reactances and one inductive reactance in the tank circuit i.e. feedback network.

- **Transistorized Colpitt's Oscillator;**

The amplifier stage uses a transistor in common emitter configuration as an active device which causes a phase shift of 180^0 and LC feedback network consisting of one inductor L and two capacitors $C_1 \, and \, C_2$, which adds further 180^0 phase shift to satisfy the oscillating conditions.

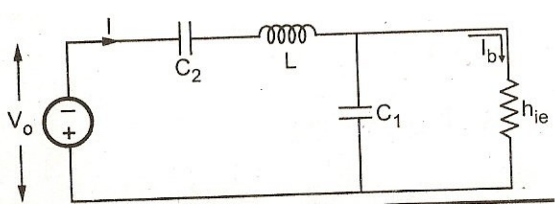


The output current $I_C = h_{fe} I_b$ acts as the input to the feedback network. While the base current I_b acts as the output current of the tank circuit, flowing through the input impedance h_{ie} of the amplifier, hence the equivalent circuit.



Derivation of frequency of oscillation

Converting the current source into voltage source we get a simplified equivalent circuit.



$V_o = h_{fe} I_b X_{C2} = h_{fe} I_b \frac{1}{j\omega C_2}$ .Total current drawn from supply $I = \frac{-V_o}{\ }$

NB: (-ve sign: Current direction is assumed opposite to voltage V_0 polarity)

$X_{C2} + X_L = \frac{1}{j\omega C_2} + j\omega L$ and $X_{C1} \| h_{ie} = \frac{\frac{1}{j\omega C_1} \times h_{ie}}{\frac{1}{j\omega C_1} + h_{ie}}$ $\Rightarrow$

$$I = \frac{-h_{fe} I_b \frac{1}{j\omega C_2}}{\left[\frac{1}{j\omega C_2} + j\omega L\right] + \left[\frac{\frac{h_{ie}}{j\omega C_1}}{h_{ie} + j\omega C_1}\right]} \quad ; \quad (i)$$

According to current division in parallel circuits $I_b = I \times \frac{X_{C1}}{X_{C1} + h_{ie}} = \frac{I \times \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + h_{ie}}$; (ii)

Substituting value of I from $eq^n(i)$ in $eq^n(ii)$ and replacing $(j\omega)^2 = -\omega 2\, ; and\, (j\omega)^3 = -j\omega^3$

$$1 = \frac{-h_{fe}}{(1 - \omega^2 LC_2) + j\omega h_{ie}[C_1 + C_2 - \omega^2 LC_1 C_2]} \quad ; \qquad (iii)$$

To satisfy Barkhausen criterion, the imaginary part of the RHS denominator must be equal to zero.

i.e. $\omega h_{ie}[C_1 + C_2 - \omega^2 LC_1 C_2] = 0 \quad \Rightarrow C_1 + C_2 - \omega^2 LC_1 C_2 = 0 \quad hence\ \omega^2 = \frac{C_1 + C_2}{LC_1 C_2}$

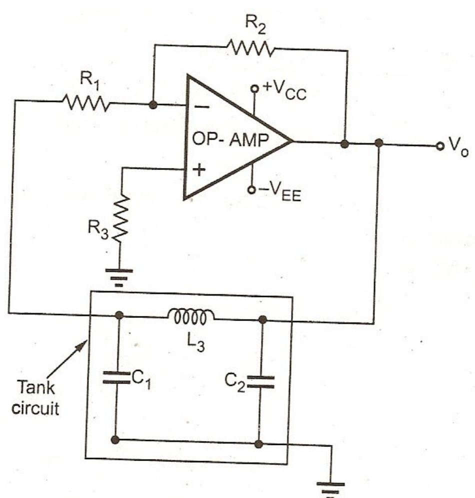
$$\omega = \frac{1}{\sqrt{LC_{eq}}} \quad \Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC_{eq}}}\ where \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$f = \frac{1}{2\pi\sqrt{LC_{eq}}}$ - Frequency of oscillation of the Colpitts oscillator.

Substituting the value of $f = \frac{1}{2\pi\sqrt{LC_{eq}}}$ in $eq^n(iii)$ and equating the magnitudes of both sides, the restriction of the value h_{fe} is obtained as, $h_{fe} = \frac{C_2}{C_1}$

- **Colpitt's Oscillator using Op-amp**

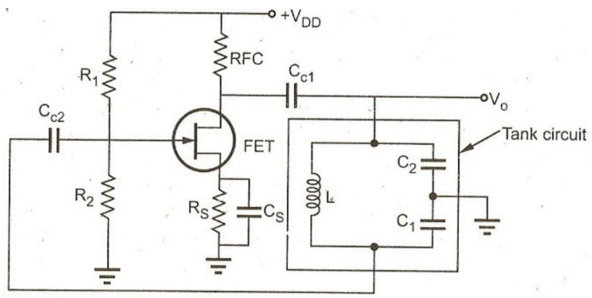
The Op-amp is used for an amplifier stage while the tank circuit remains as in a transistorized circuit.



The oscillator frequency $f = \frac{1}{2\pi\sqrt{LC_{eq}}}$; the condition of gain of the op-amp is now applicable.

- **Colpitt's Oscillator using FET**

The FET is used as an active device in the amplifier stage, the tank circuit remain the same and the oscillating frequency also remain the same.



Example:

Find the frequency of oscillation of a transistorized Colpitts oscillator having

$C_1 = 150\ pF,\ C_2 = 1.5\,nF\ and\ L = 50\,\mu H$

Solution

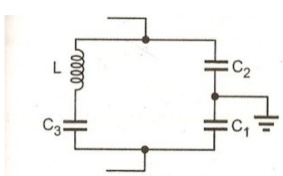
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{150\times10^{-12}\times1.5\times10^{-9}}{150\times10^{-12}+1.5\times10^{-9}} = 136.363\ pF$$

$$f = \frac{1}{2\pi\sqrt{LC_{eq}}} = \frac{1}{2\pi\sqrt{150\times10^{-12}\times136.363\times10^{-12}}} = 1.927\ MHz$$

5. Analysis of Clapp and Armstrong Oscillators,

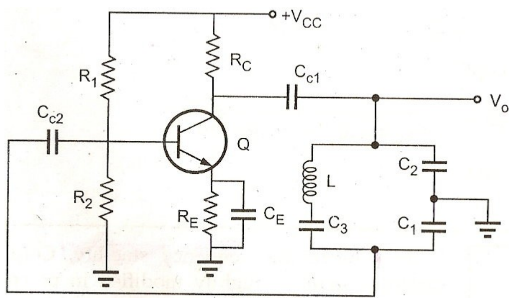
A. Analysis of Clapp Oscillator;

To achieve the frequency stability, the Colpitts oscillator circuit is slightly modified to get Clapp oscillator such that the basic tank circuit with two capacitive reactances and one inductive reactance remains the same, but one more capacitor C_3 is introduced in series with the inductance L .



The value of C_3 is much smaller than the values of C_1 and C_2 , hence the equivalent capacitance becomes; $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \approx \frac{1}{C_3}$. In practice the values C_1 and C_2 are neglected thus $C_3 = C_{eq}$ and the frequency of oscillation is given by $f = \frac{1}{2\pi\sqrt{LC_3}}$,

A transistorized Clapp oscillator



There is no transistor parameter, like stray capacitance across C_3, hence the frequency is stable and accurate,

- The frequency can be varied in the desired range;

  C_3 can be kept variable, and since the frequency depends on C_3, the frequency can be varied in the desired range

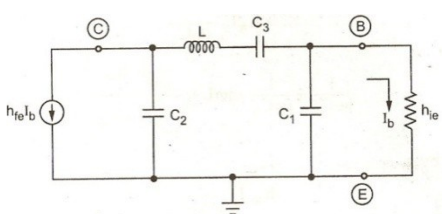
Advantages of the Clapp oscillator;

- Frequency stability;

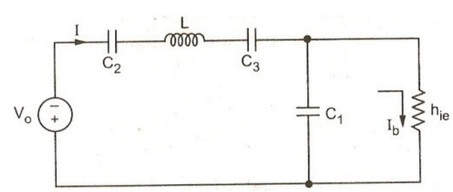
Derivation of frequency of oscillations

Derivation is similar to the Colpitts oscillator with C_3 in series with L in the equivalent

circuit of the transistorized Clapp oscillator.



Equivalent circuit



Simplified equivalent circuit with current source converted to voltage source

$$V_o = h_{fe} I_b X_{C2} = h_{fe} I_b \frac{1}{j\omega C_2}$$ Total current drawn from supply

$$I = \frac{-V_o}{\left[X_{C2} + X_{C3} + X_L\right] + \left[X_{C1} \| h_{ie}\right]}$$

NB: (-ve sign: Current direction is assumed opposite to voltage V_0 polarity)

$$X_{C2} + X_{C3} + X_L = \frac{1}{j\omega C_2} + \frac{1}{j\omega C_3} + j\omega L$$ and $$X_{C1} \| h_{ie} = \frac{\frac{1}{j\omega C_1} \times h_{ie}}{\frac{1}{j\omega C_1} + h_{ie}}$$

Therefore $$I=\frac{-h_{fe}I_b\frac{1}{j\omega C_2}}{\left[\frac{1}{j\omega C_2}+\frac{1}{j\omega C_3}+j\omega L\right]+\left[\frac{\frac{h_{ie}}{j\omega C_1}}{h_{ie}+j\omega C_1}\right]}\quad;\quad(i)$$

According to current division in parallel circuits $$I_b=I\times\frac{X_{C1}}{X_{C1}+h_{ie}}=\frac{I\times\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}+h_{ie}}\quad;\quad(ii)$$

Substituting value of I from $eq^n(i)$ in $eq^n(ii)$ and replacing $(j\omega)^2=-\omega 2\,;\,and\,(j\omega)^3=-j\omega^3$

$$1=\frac{-h_{fe}C_3}{(C_2C_3-\omega^2LC_2C_3)+j\omega h_{ie}\left[(C_1C_2+C_2C_3+C_3C_1)-\omega^2LC_1C_2C_3\right]}\quad;\qquad(iii)$$

To satisfy Barkhausen criterion, the imaginary part of the RHS denominator must be equal to zero.

i.e. $\omega h_{ie}\left[C_1C_2+C_2C_3+C_3C_1-\omega^2LC_1C_2C_3\right]=0 \;\Rightarrow C_1C_2+C_2C_3+C_3C_1-\omega^2LC_1C_2C_3=0$

$hence\,\omega^2=\frac{C_1C_2+C_2C_3+C_3C_1}{LC_1C_2C_3}$. Therefore $\omega^2=\frac{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}{L}=\frac{\frac{1}{C_{eq}}}{L}$

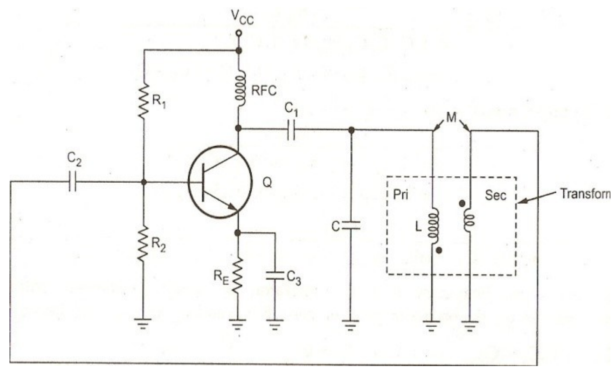
$\omega^2=\frac{1}{LC_{eq}}\Rightarrow\omega=\frac{1}{\sqrt{LC_{eq}}}\quad Hence\quad f=\frac{1}{2\pi\sqrt{LC_{eq}}}$ But as $C_3<<C_1;\,and\,C_3<<C_2\Rightarrow C_{eq}=C_3;$

$Hence\;f=\frac{1}{2\pi\sqrt{LC_3}}$ The required frequency of oscillation of the Clapp oscillator

B. Analysis of Armstrong Oscillator;

In the Armstrong Oscillator a transformer is used, whose primary winding acts as L in the circuit while the voltage across secondary winding is used as a feedback.

C_1 and C_2 are coupling capacitors, the collector of transistor Q drives the LC resonating circuit with primary of transformer acting as L. The feedback signal is taken from the secondary winding which is small and given to the base of the transistor Q.



There is mutual inductance M between primary and secondary of the transformer which introduces a phase shift of 180^0, as indicated by the dots, in addition to the phase shift of 180^0 by transistor Q. Thus overall phase shift around the loop is 360^0 which satisfies the Barkhausen criterion.

The feedback fraction $\beta = \frac{M}{L}$ For oscillations to start $\beta A > 1$, hence $\frac{1}{\beta} > \frac{L}{M}$ or $A_{min} = \frac{L}{M}$.

The frequency of sustained oscillations $f_r = \frac{1}{2\pi\sqrt{LC}}$

Disadvantages:

- The frequency of sustained oscillations is dependent on primary winding inductance L and C across it,
- Transformers are generally avoided due to their size and cost.

Frequency range of RC and LC oscillators

RC oscillators are widely used in audio frequency range

LC oscillators are commonly used higher radio frequency range

6. Frequency Stability of Oscillators;

Meaning;

- Frequency stability of an oscillator is the measure of the ability of an oscillator to maintain the desired frequency as precisely as possible for as long a time as possible

Sources of instability (or factors that affect the frequency stability of an oscillator);

- In a transistorized Colpitt's and Hartley oscillator, the base-collector junction is reverse biased, and there exists an internal capacitance which is dominant at high frequencies. This capacitive effect in transistor and stray capacitances affect the value of capacitance in the tank circuit and hence the oscillating frequency.

- Tank circuit component parameters are temperature sensitive. Changes in values of inductors and capacitors due to changes in temperature are the main cause due to which frequency does not remain stable.
- Active device parameters such as BJT, FET are temperature sensitive. As temperature changes the parameters get affected in-turn affect the oscillating frequency.
- Variation in power supply,
- Changes in atmospheric conditions, aging and unstable transistor parameters,
- Changes in the load connected, affect the effective resistance of the tank circuit,

Methods of improving the frequency stability

The frequency stability of an oscillator can be improved by the following modifications:

- Enclosing the circuit in a constant temperature chamber,
- Maintaining constant voltage by using Zener diodes,
- Couple the oscillator loosely to reduce the load effect.