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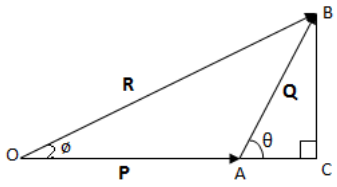
Triangle Law of Vector Addition

Statement of Triangle Law

If 2 vectors acting simultaneously on a body are represented both in magnitude and direction by 2 sides of a triangle taken in an order then the resultant(both magnitude and direction) of these vectors is given by 3rd side of that triangle taken in opposite order.

Derivation of the law

Consider two vectors **P** and **Q** acting on a body and represented both in magnitude and direction by sides OA and AB respectively of a triangle OAB. Let θ be the angle between **P** and **Q**. Let **R** be the resultant of vectors **P** and **Q**. Then, according to triangle law of vector addition, side OB represents the resultant of **P** and **Q**.



So, we have

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

Now, expand A to C and draw BC perpendicular to OC.

From triangle OCB,

$$OB^2 = OC^2 + BC^2$$
$$or, OB^2 = (OA + AC)^2 + BC^2 \quad .....(i)$$

In triangle ACB,

$$\cos\theta = \frac{AC}{AB}$$
$$or, AC = AB\cos\theta = Q\cos\theta$$

Also,

$$\sin\theta = \frac{BC}{AB}$$
$$or, BC = AB\sin\theta = Q\sin\theta$$

Magnitude of resultant:

Substituting value of AC and BC in (i), we get

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$
$$or, R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$
$$or, R^2 = P^2 + 2PQ\cos\theta + Q^2$$
$$\therefore R = \sqrt{P^2 + 2PQ\cos\theta + Q^2}$$

which is the magnitude of resultant.

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Direction of resultant: Let ø be the angle made by resultant **R** with **P**. Then,

From triangle OBC,

$$\tan\phi = \frac{BC}{OC} = \frac{BC}{OA + AC}$$
$$or, \tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta}$$
$$\therefore \phi = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

which is the direction of resultant.

Submitted by Sagun Shrestha

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