UNIT-II NETWORK THEOREM

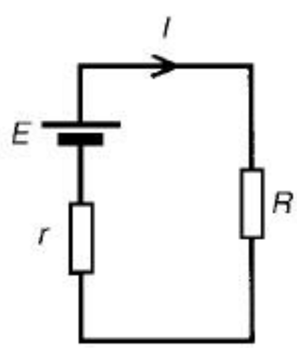
THEVENINS THEOREM:

In circuit theory, **Thévenin's theorem** for linear electrical networks states that any combination of voltage sources, current sources, and resistors with two terminals is electrically equivalent to a single voltage source V and a single series resistor R. For single frequency AC systems the theorem can also be applied to general impedances, not just resistors.

The procedure adopted when using Théveni determine the current in any branch of an active network (i.e. one containing a source of e.m.f.):

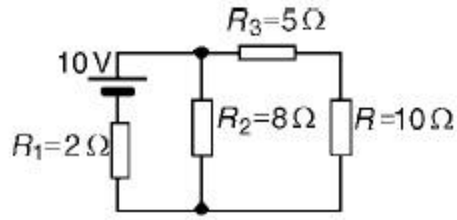
(i) remove the resistance R from that branch,
(ii) determine the open-circuit voltage, E, across the break,

remove each source of e.m.f. and replace them by their internal resistances and then determine the resistance, r, 'looking-in'break.
(iv) determine the value of the current from the equivalent circuit shown in Figure 13.33, i.e. **I = ER+r**

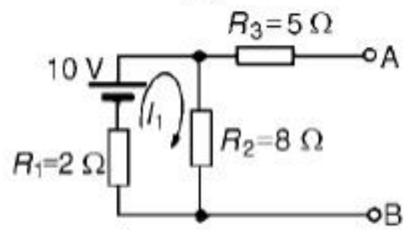


Problem 1: Use Thévenin's theorem to find the current flowing in the 10 Ω resistor for the circuit shown in Figure

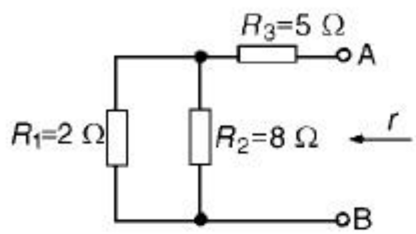
Following the above procedure:
The 10 Ωresistance is removed from the circuit as shown in Figure There is no current flowing in the 5 Ωresistor and current I1 is given by:
I1 = 10/R1 + R2
= 10/2 + 8
= 1A



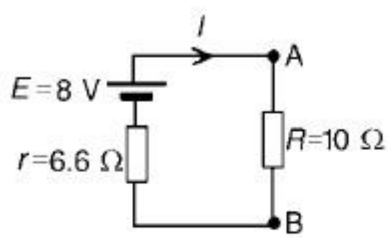
(a)



(b)



(c)



(d)

P.d. across R2 =I1R2 =1×8=8V Hence p.d. across AB, i.e. the open-circuit voltage across the break, E =8V

(iii) Removing the source of e.m.f. gives the circuit of Figure Resistance, r = R3 + R1R2/R1 + R2

=5+ (2×8/2+8)

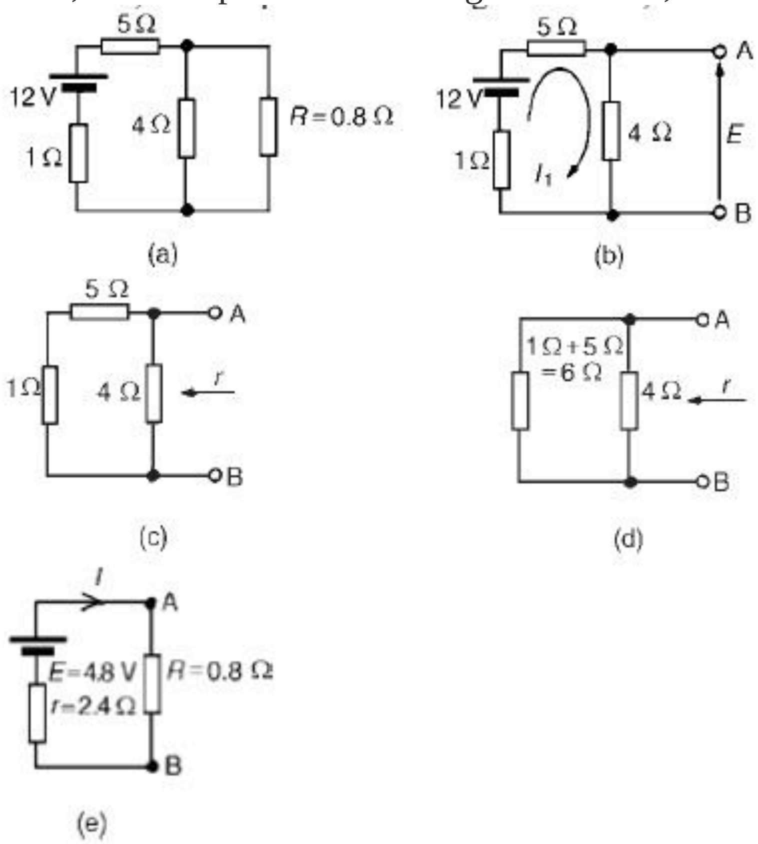
= 5 + 1.6 = 6.6 Ω

(iv) The equivalent Thévenin's circuit is shown in F

Current I = E/R+r= 8/10+6.6= 8/16.6=0.482A

Problem 2: For the network shown in Figure determine the current in the 0.8 Ω resistor using
Thévenin's theorem.

Following the procedure:
The 0.8_ resistor is removed from the circuit as shown in Figure
Current I1 = 12/1+5+4= 12/10
=1.2A
P.d. across 4 Ωresistor=4I1 =(4) (1.2)=4.8V
Hence p.d. across AB, i.e. the open-circuit voltage across AB, E =4.8V



(iii) Removing the source of e.m.f. gives the circuit shown in Figure (c). The equivalent circuit of Figure

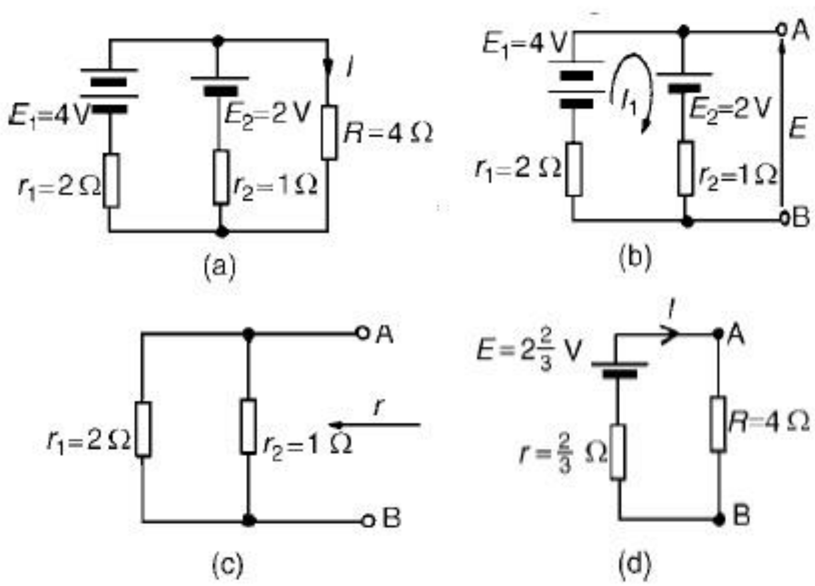
(c) is shown in Figure (d), from which,resistance r = 4×6/4+6= 24/10
=2.4 Ω
(iv) The equivalent Thévenin's circuit I=E/+R is show
=4.8/2.4+0.8
=4.8/3.2

I = 1.5A=current in the 0.8 Ωresistor

Problem 3: U Use Thévenin’s theorem to determine the current I flowing in the 4 Ω resistor shown in Figure.

Find also the power dissipated in the 4 Ωresistor.

(i) The 4 Ωresistor is removed from the circuit as shown in Figure



(ii) Current I1 = E1 −E2/r1 +r2

=4−2/2+1

=2/3A

P.d. across AB, $E = E_1 - I_1 r_1 = 4 - \left(\frac{2}{3}\right)(2) = 2\frac{2}{3}$ V

(iii) Removing the sources of e.m.f. gives the circuit shown in Figure (c), from which resistance

r = 2 × 1/2 + 1 = 2/3 Ω

(iv)The equivalent Thévenin’s circuit is show

$$\text{current, } I = \frac{E}{r+R} = \frac{2\frac{2}{3}}{\frac{2}{3}+4} = \frac{8/3}{14/3}$$

=8/14

=0.571A

= current in the 4 Ωresistor

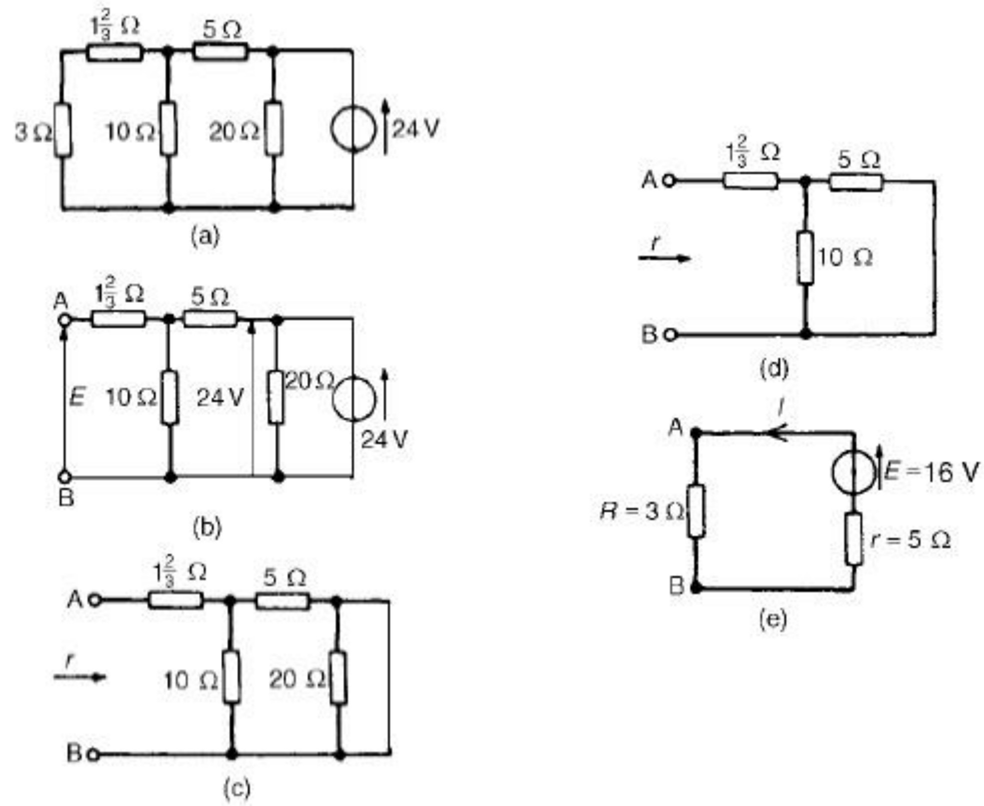
Problem 4: Power dissipated in 4 Ωresistor,P=I2R=(0.571)2 (4)=1.304W Use Thévenin's t to determine the current flowing in the 3 Ω resistance of the network shown in Figure (a). The

voltage source has negligible internal resistance.
i) The 3 Ωresistance is removed from the circuit as shown in Figure (b).

(ii) The 1 2/3 Ωresistance now carries no current. P.d. across 10 Ωresistor=(10/10+5)(24)

=16V
Hence p.d. across AB, E =16V

(iii) Removing the source of e.m.f. and replacing it by its internal resistance means that the 20 Ωresistance is short-circuited as shown in Figure (c) since its internal resistance is zero. The 20 Ωresistance may thus be removed as shown in Figure (d)

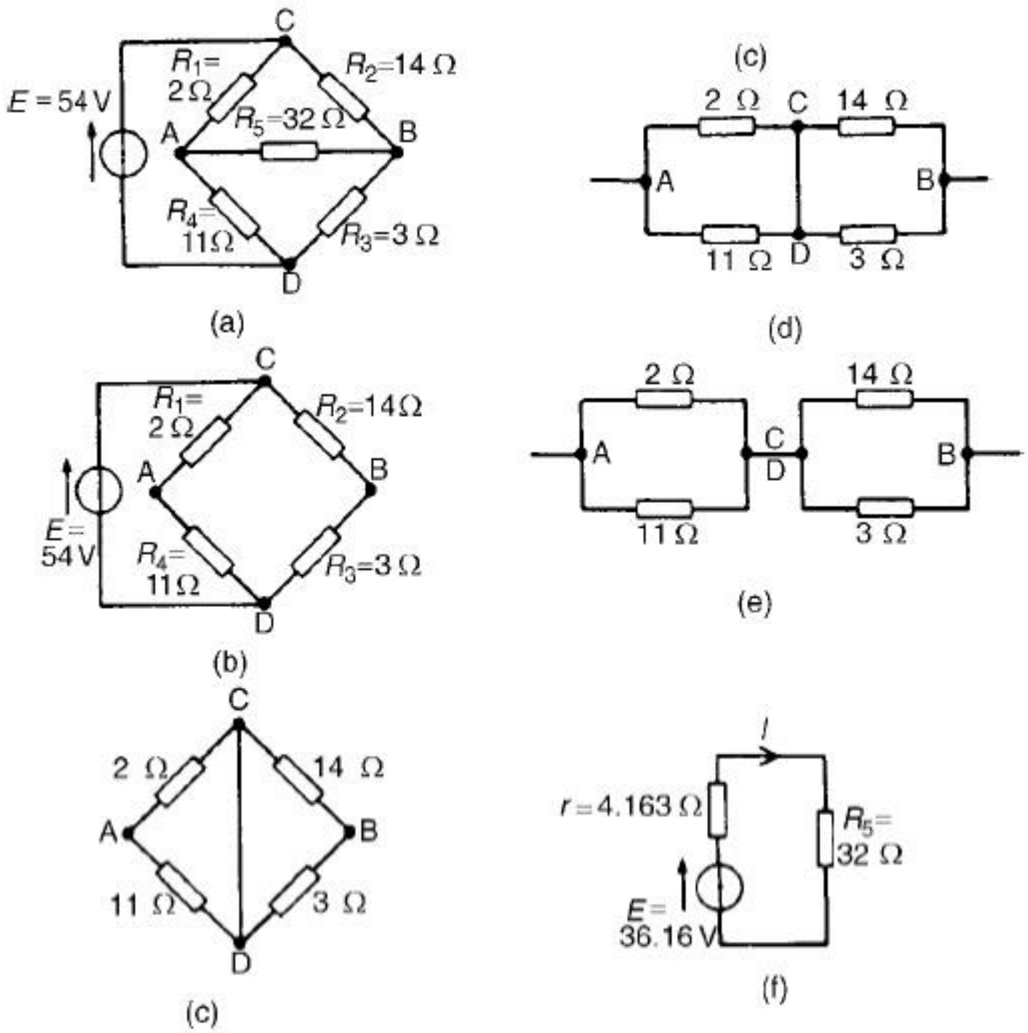


From Figure (d), resistance,
r =1 2/3+ 10×5/10+5 =5 Ω

(iv) The equivalent Thévenin's circuit is show current, I = E/r +R= 16/3+5= 16/8

=2A

Problem 5: AWheatstone Bridge network is shown in Figure (a). Calculate the current flowing in the 32 Ωresistor, and its direction, using Théveni negligible resistance.



The 32 Ωresistor is removed from the circuit as shown in Figure (b) The p.d. between A and C,

VAC =R1/R1 +R4 (E) =2/2+11(54) = 8.31V The p.d. between B and C,

VBC =R2/R2 +R3 (E) =14/14+3(54) = 44.47V Hence the p.d. between A and B=44.47−8.31=36.16V

Point C is at a potential of +54V. Between C and A is a voltage drop of 8.31V. Hence the voltage at point A is 54−8.31=45.69V. Between C and B is a voltage drop of 44.47V. Hence the voltage at point B is 54−44.47=9.53V. Since the voltage at A is greater than at B, current must flow in the direction A to B.

Replacing the source of e.m.f. with a short-circuit (i.e. zero internal resistance) gives the circuit shown in Figure (c). The circuit is redrawn and simplified as shown in Figure (d) and (e), from which the resistance between terminals A and B,

r = 2 × 11/2 + 11+ 14 × 3/14 + 3

= 22/13+ 42/17

= 1.692 + 2.471 = 4.163 Ω

(iv) The equivalent Thévenin’s circuit is show

current I = E/r +R5

= 36.16/4.163+32 =1A

NORTON’S THEOREM:

Norton’s theorem states the following:

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current and a parallel resistor.

The steps leading to the proper values of IN and RN. Preliminary steps:

1. Remove that portion of the network across which the Norton equivalent circuit is found.
2. Mark the terminals of the remaining two-terminal network.
3. Finding RN:

Calculate RN by first setting all sources to zero and then finding the resultant resistance between the two marked terminals. Since RN = RTh the procedure and value obtained using the approach described for Thevenin’s theorem will determine the proper value of RN.

4. Finding IN :

Calculate IN by first returning all the sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Problem 1: Use Norton's theorem to determine Ω resistance for the the cur circuit shown in Figure

The branch containing the 10 Ω resistance is short circuited as shown in Figure Figure (c) is equivalent to Figure (b).Hence *ISC* = 10/2

= 5A

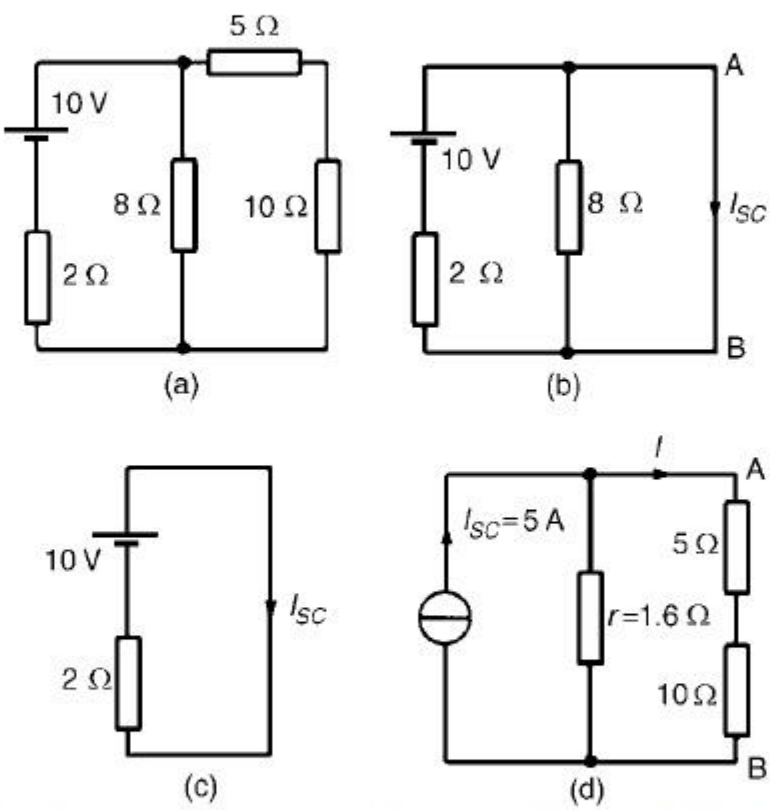
If the 10V source of e.m.f. is removed from Figure (b) the resistance 'looking-in' at a break made between A and B is given by:

$r = 2 \times 8/2 + 8 = 1.6\ \Omega$

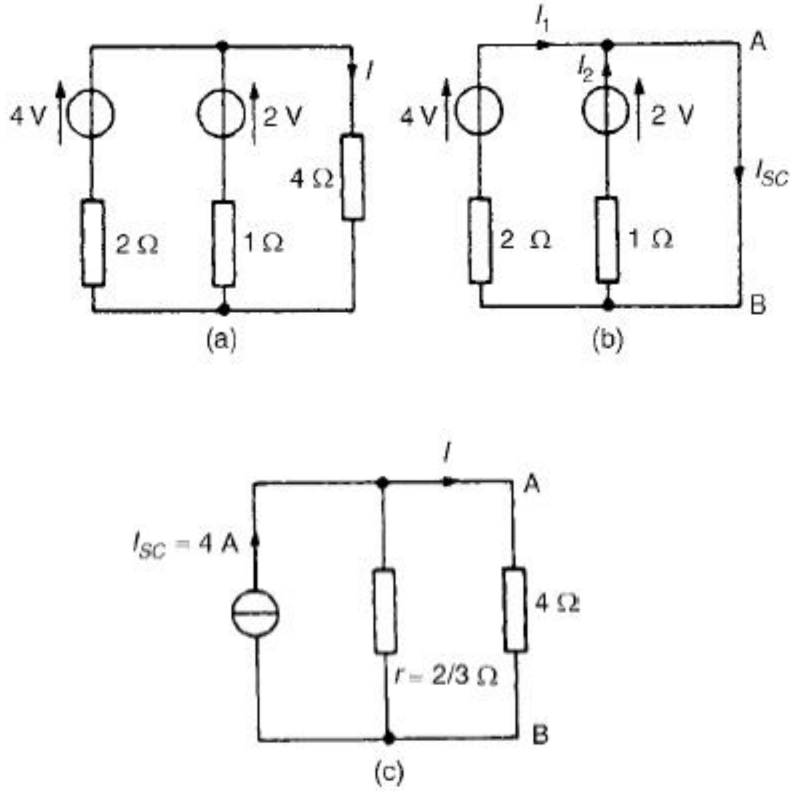
From the Norton equivalent network shown in Figure(d) the current in the 10 Ωresistance, by current division, is given by:

$I = (1.6/1.6 + 5 + 10)\ (5) = 0.482\text{A}$

as obtained previously in problem 7 using Thévenin's theorem.



Problem 2: Problem 2: Use Norton's theorem to determine the current I flowing in the 4 Ω resistance shown in Figure (a).



The 4 Ωbranch is short-circuited as shown in Figure (b). From Figure (b), $I_{SC} = I_1 + I_2 = 4$A

If the sources of e.m.f. are removed the resistance 'looking-in'at a break made be given by:

$r = 2 \times 1/2 + 1 = 2/3$ Ω

From the Norton equivalent network shown in Figure (c) the current in the 4 Ωresistance is given by:

$I = (2/3/(2/3) + 4)(4) = 0.571$A,

Superposition theorem

Superposition theorem is based on the concept of linearity between the response and excitation of an electrical circuit. It states that the response in a particular branch of a linear circuit when multiple independent sources are acting at the same time is equivalent to the sum of the responses due to each independent source acting at a time.

In this method, we will consider only **one independent source** at a time. So, we have to eliminate the remaining independent sources from the circuit. We can eliminate the voltage sources by shorting their two terminals and similarly, the current sources by opening their two terminals.

Therefore, we need to find the response in a particular branch **'n' times** if there are 'n' independent sources. The response in a particular branch could be either current flowing through that branch or voltage across that branch.

Procedure of Superposition Theorem

Follow these steps in order to find the response in a particular branch using superposition theorem.

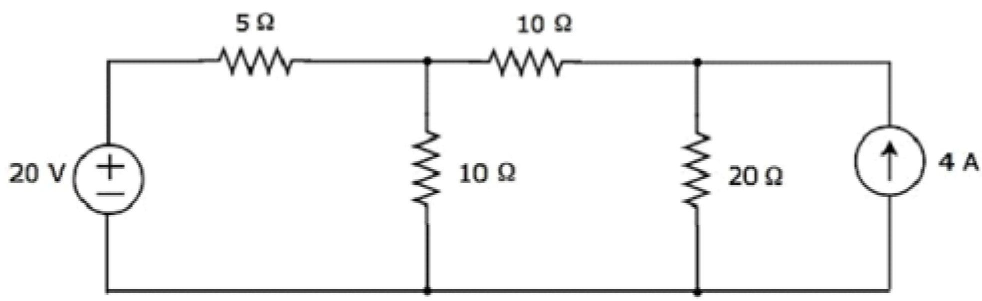
Step 1 − Find the response in a particular branch by considering one independent source and eliminating the remaining independent sources present in the network.

Step 2 − Repeat Step 1 for all independent sources present in the network.

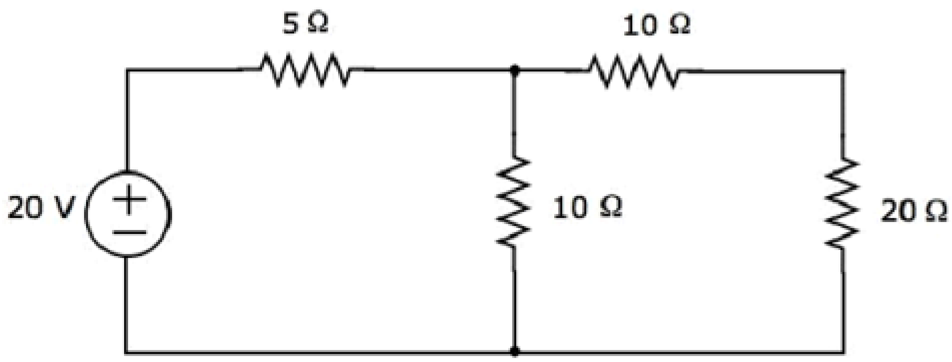
Step 3 − Add all the responses in order to get the overall response in a particular branch when all independent sources are present in the network.

Example

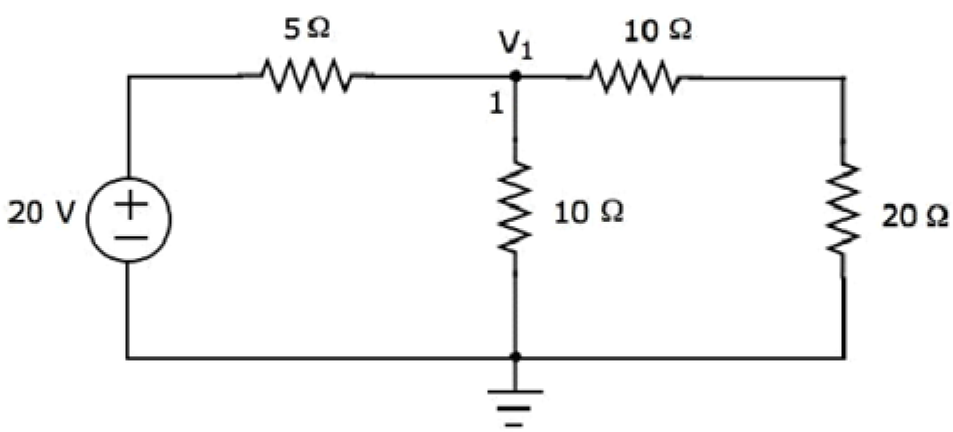
Find the current flowing through 20 Ω resistor of the following circuit using **superposition theorem**.



Step 1 − Let us find the current flowing through 20 Ω resistor by considering only **20 V voltage source**. In this case, we can eliminate the 4 A current source by making open circuit of it. The modified circuit diagram is shown in the following figure.



There is only one principal node except Ground in the above circuit. So, we can use **nodal analysis** method. The node voltage V_1 is labelled in the following figure. Here, V_1 is the voltage from node 1 with respect to ground.



The **nodal equation** at node 1 is

V1−205+V110+V110+20=0V1−205+V110+V110+20=0

⇒6V1−120+3V1+V130=0⇒6V1−120+3V1+V130=0

⇒10V1=120⇒10V1=120

⇒V1=12V⇒V1=12V

The **current flowing through 20 Ω resistor** can be found by doing the following simplification.

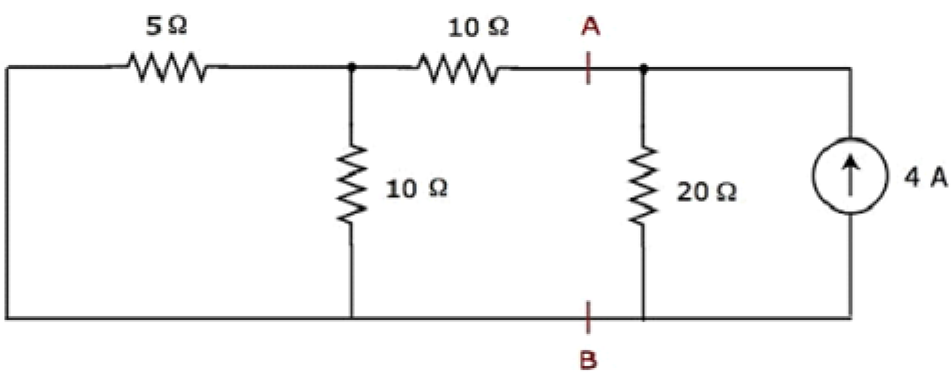
I1=V110+20I1=V110+20

Substitute the value of V_1 in the above equation.

I1=1210+20=1230=0.4AI1=1210+20=1230=0.4A

Therefore, the current flowing through 20 Ω resistor is **0.4 A**, when only 20 V voltage source is considered.

Step 2 − Let us find the current flowing through 20 Ω resistor by considering only **4 A current source**. In this case, we can eliminate the 20 V voltage source by making short-circuit of it. The modified circuit diagram is shown in the following figure.

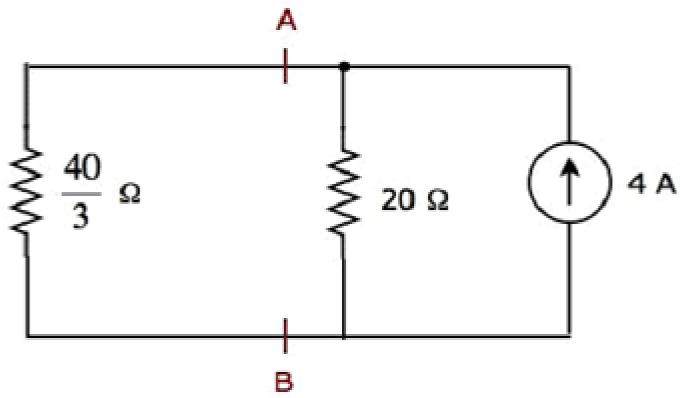


In the above circuit, there are three resistors to the left of terminals A & B. We can replace these resistors with a single **equivalent resistor**. Here, 5 Ω & 10 Ω resistors are connected in parallel and the entire combination is in series with 10 Ω resistor.

The **equivalent resistance** to the left of terminals A & B will be

RAB=(5×105+10□+10=103+10=403ΩRAB=(5×105+10□+10=103+10=403Ω

The simplified circuit diagram is shown in the following figure.



We can find the current flowing through 20 Ω resistor, by using **current division principle**.

I2=IS(R1R1+R2□I2=IS(R1R1+R2□

Substitute IS=4A,R1=403ΩIS=4A,R1=403Ω and R2=20ΩR2=20Ω in the above equation.

I2=4(403403+20□=4(40100□=1.6AI2=4(403403+20□=4(40100□=1.6A

Therefore, the current flowing through 20 Ω resistor is **1.6 A**, when only 4 A current source is considered.

Step 3 − We will get the current flowing through 20 Ω resistor of the given circuit by doing the **addition of two currents** that we got in step 1 and step 2. Mathematically, it can be written as

I=I1+I2I=I1+I2

Substitute, the values of I_1 and I_2 in the above equation.

I=0.4+1.6=2AI=0.4+1.6=2A

Therefore, the current flowing through 20 Ω resistor of given circuit is **2 A**.

Maximum power transfer theorem

Maximum power transfer theorem states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.

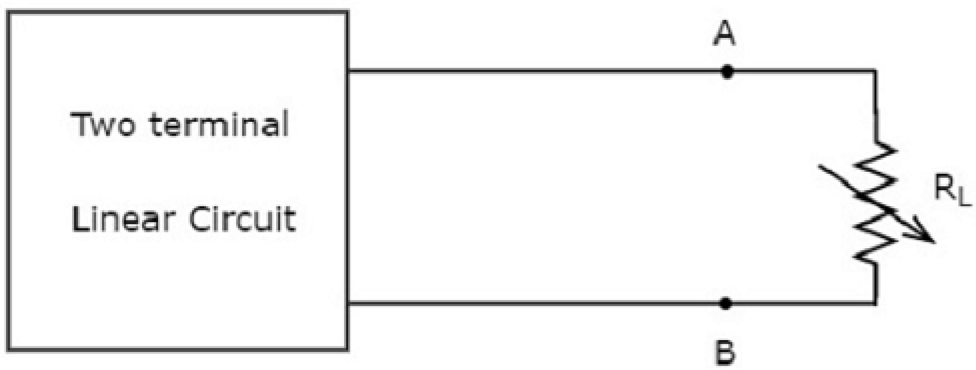
Similarly, **Maximum power transfer theorem** states that the AC voltage source will deliver maximum power to the variable complex load only when the load impedance is equal to the complex conjugate of source impedance.

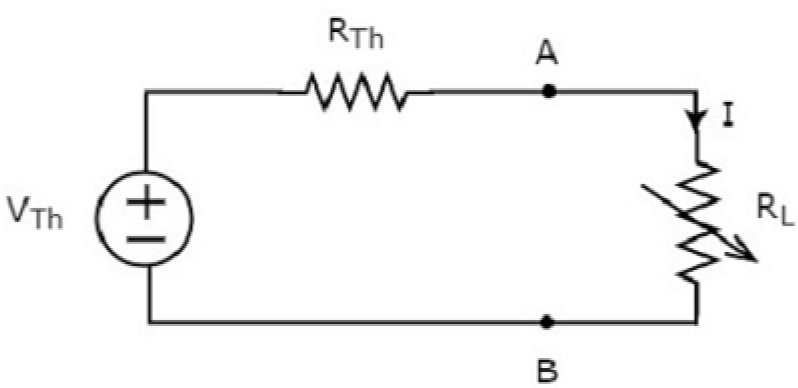
In this chapter, let us discuss about the maximum power transfer theorem for DC circuits.

Proof of Maximum Power Transfer Theorem

Replace any two terminal linear network or circuit to the left side of variable load resistor having resistance of R_L ohms with a Thevenin's equivalent circuit. We know that Thevenin's equivalent circuit resembles a practical voltage source.

This concept is illustrated in following figures.





The amount of power dissipated across the load resistor is

PL=I2RLPL=I2RL

Substitute I=VThRTh+RLI=VThRTh+RL in the above equation.

PL=(VTh(RTh+RL)□2RLPL=(VTh(RTh+RL)□2RL

⇒PL=VTh2{RL(RTh+RL)2}⇒PL=VTh2{RL(RTh+RL)2} **Equation 1**

Condition for Maximum Power Transfer

For maximum or minimum, first derivative will be zero. So, differentiate Equation 1 with respect to R_L and make it equal to zero.

dPLdRL=VTh2{(RTh+RL)2×1−RL×2(RTh+RL)(RTh+RL)4}=0dPLdRL=VTh2{(RTh+RL)2×1−RL×2(RTh+RL)(RTh+RL)4}=0

⇒(RTh+RL)2−2RL(RTh+RL)=0⇒(RTh+RL)2−2RL(RTh+RL)=0

⇒(RTh+RL)(RTh+RL−2RL)=0⇒(RTh+RL)(RTh+RL−2RL)=0

⇒(RTh−RL)=0⇒(RTh−RL)=0

⇒RTh=RLorRL=RTh⇒RTh=RLorRL=RTh

Therefore, the **condition for maximum power** dissipation across the load is RL=RThRL=RTh. That means, if the value of load resistance is equal to the value of source resistance i.e., Thevenin's resistance, then the power dissipated across the load will be of maximum value.

The value of Maximum Power Transfer

Substitute RL=RTh&PL=PL,MaxRL=RTh&PL=PL,Max in Equation 1.

PL,Max=VTh2{RTh(RTh+RTh)2}PL,Max=VTh2{RTh(RTh+RTh)2}

PL,Max=VTh2{RTh4RTh2}PL,Max=VTh2{RTh4RTh2}

⇒PL,Max=VTh24RTh⇒PL,Max=VTh24RTh

⇒PL,Max=VTh24RL,sinceRL=RTh⇒PL,Max=VTh24RL,sinceRL=RTh

Therefore, the **maximum amount of power** transferred to the load is

PL,Max=VTh24RL=VTh24RThPL,Max=VTh24RL=VTh24RTh

Efficiency of Maximum Power Transfer

We can calculate the efficiency of maximum power transfer, ηMaxηMax using following formula.
ηMax=PL,MaxPSηMax=PL,MaxPS **Equation 2**

Where,

- PL,MaxPL,Max is the maximum amount of power transferred to the load.
- PSPS is the amount of power generated by the source.

The **amount of power generated** by the source is

PS=I2RTh+I2RLPS=I2RTh+I2RL

⇒PS=2I2RTh,sinceRL=RTh⇒PS=2I2RTh,sinceRL=RTh

- Substitute I=VTh2RThI=VTh2RTh in the above equation.

PS=2(VTh2RTh□2RThPS=2(VTh2RTh□2RTh

⇒PS=2(VTh24RTh2□RTh⇒PS=2(VTh24RTh2□RTh

⇒PS=VTh22RTh⇒PS=VTh22RTh

- Substitute the values of PL,MaxPL,Max and PSPS in Equation 2.

ηMax=(VTh24RTh□(VTh22RTh□ηMax=(VTh24RTh□(VTh22RTh□

⇒ηMax=12⇒ηMax=12

We can represent the efficiency of maximum power transfer in terms of **percentage** as follows −

%ηMax=ηMax×100%%ηMax=ηMax×100%

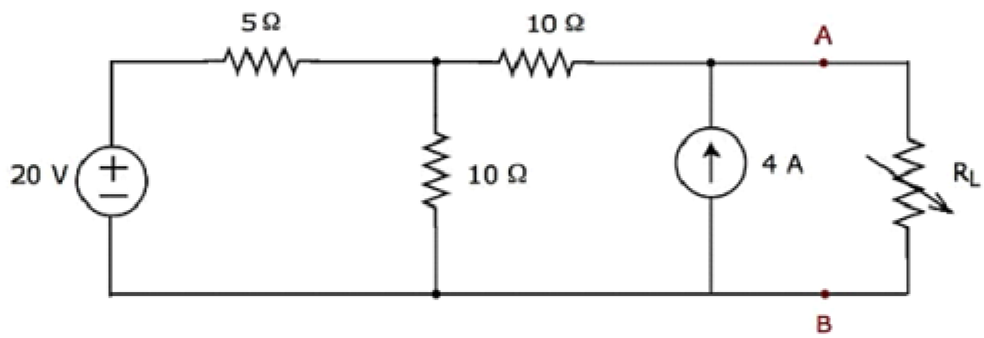
⇒%ηMax=(12□×100%⇒%ηMax=(12□×100%

⇒%ηMax=50%⇒%ηMax=50%

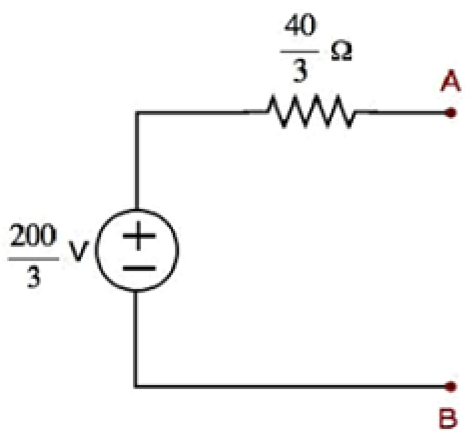
Therefore, the efficiency of maximum power transfer is **50 %**.

Example

Find the **maximum power** that can be delivered to the load resistor R_L of the circuit shown in the following figure.

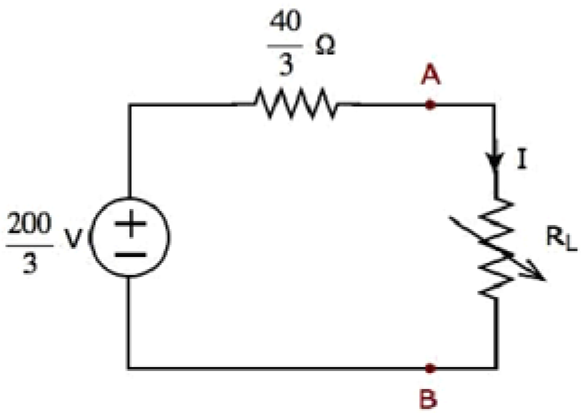


Step 1 − In Thevenin's Theorem chapter, we calculated the Thevenin's equivalent circuit to the left side of terminals A & B. We can use this circuit now. It is shown in the following figure.



Here, Thevenin's voltage VTh=2003VVTh=2003V and Thevenin's resistance RTh=403ΩRTh=403Ω

Step 2 − Replace the part of the circuit, which is left side of terminals A & B of the given circuit with the above Thevenin's equivalent circuit. The resultant circuit diagram is shown in the following figure.



Step 3 − We can find the maximum power that will be delivered to the load resistor, R_L by using the following formula.

PL,Max=VTh24RThPL,Max=VTh24RTh

Substitute VTh=2003VVTh=2003V and RTh=403ΩRTh=403Ω in the above formula.

PL,Max=(2003□24(403□PL,Max=(2003□24(403□

PL,Max=2503WPL,Max=2503W

Therefore, the **maximum power** that will be delivered to the load resistor RL of the given circuit is 25032503 **W**