052 ME 41 Dynamics of Machines

Unit I: Force Analysis

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Contents ..

Unit I: Force Analysis

- Rigid Body dynamics in general plane motion – Equations of motion
- Dynamic force analysis - Inertia force and Inertia torque – D'Alemberts principle - The principle of superposition
- Dynamic Analysis in Reciprocating Engines – Gas Forces - Equivalent masses - Bearing loads -
- Crank shaft Torque - Turning moment diagrams - Fly wheels –Engine shaking Forces –
- Cam dynamics - Unbalance, Spring, Surge and Windup.

Learning Objectives...

After the series of lecture sessions of this unit, the students should be able to :

- Analyze forces and torques required to produce motions in mechanisms.
- Evaluate forces acting on the machine members or mechanisms at a particular configuration.
- Apply force analysis techniques to design machines, mechanisms, and machine elements subjected to static and dynamic forces

Lesson 1

Basics of Force Analysis

Basics …

Force Analysis

- ***Law I.*** A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.
- ***Law II.*** The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.*
- ***Law III.*** The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

- Force is a push or pull, which acts on a body changes or tends to change the state of rest or of uniform motion of the body.
- A force is completely characterized by its point of application, its magnitude and direction
- Newton's second law forms the basis for most of the analysis in dynamics & helps us to measure force quantitatively

$$a \sim F$$
$$a \sim 1/m$$

Basics …

- Newtonian mechanics (branch of mechanics based on Newton's Laws):
  - Conservation of momentum $\vec{F} = 0 \Rightarrow \vec{a} = 0$
  - Force = rate of change of momentum $\vec{F} = \frac{d}{dt}\left(m\vec{v}\right)$
  - Action/reaction $\vec{F}_{12} = \vec{F}_{21}$

- Corollary (proposition formed based on Newton's Laws), Euler's Equation:
  - Torque = rate of change of angular momentum

$$\vec{T} = \frac{d}{dt}\left(I\omega\right) = I\alpha + \omega \times I\omega$$

Basics …

Types of forces …

- **Applied forces:**
  - The external force acting on a system of body from outside the system are called applied force. The applied forces are classified as active and reactive force
  - Examples: gas, hydraulic & pneumatic forces etc., electrical, magnetic & gravity forces.
- **Inertia forces:**
  - Due to mass of accelerating parts $(F=m.a)$.
- **Friction forces:**
  - Due to friction $(F_f=\mu F_N)$.
- **Constraint forces:**
  - Restrict the freedom of movement of objects, e.g., reaction forces, forces acting on joints.

Basics …

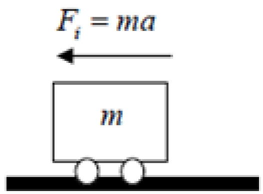
Force analysis in mechanisms

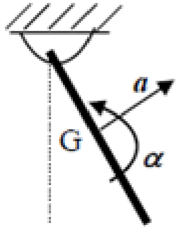
- The objective of force analysis is to find/analyze the transformation of forces, e.g., from the input to the output links (… this transformation of forces depends on the position of the mechanism; in other words, it is a function of time/Thus, it is important to find how forces change during one cycle).
- One should differentiate between external and internal forces.
  - The former are forces that are applied to the links from external sources — driving forces, resistance forces, etc.
  - The latter are forces acting between the joints (they are called constraint or reaction forces).

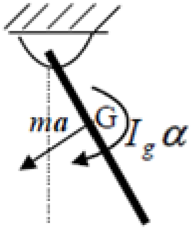
Basics …

Force analysis in mechanisms

- In force analysis, we frequently refer to two kind of forces:
  - static forces, and
  - inertial forces (due to masses of accelerating parts)
- Inertial forces are also known as d'Alembert's forces
- Static forces refer to all other forces except inertia forces.



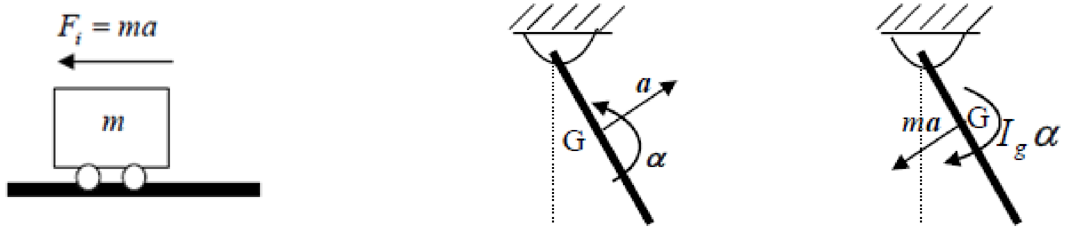




Basics …

Static vs. Dynamic Force analysis

- Dynamic analysis: When inertia forces are considered in the analysis of the mechanism, the analysis is known as dynamic force analysis.



- Static analysis: When static forces only are considered in the analysis of the mechanism, the analysis is known as dynamic force analysis (inertia forces are NOT considered)

Lesson 1 Revision Problems

1) Define the following terms: (a) Force, (b) Applied Force (c) Inertia Force (d) Torque (e) D'Alembert's Forces (f) Static Forces

2) What is Newtonian Mechanics? State the Newton's Laws applied to force analysis?

3) What are the differences between static force analysis and dynamic force analysis?

4) Differentiate between external and internal forces.

End…

Any Questions?

Lesson 2

Force Analysis Techniques

Force Analysis Techniques …

- **Three commonly used techniques**
  - Superposition
  - Matrix method
  - Virtual Work (Energy) method

- **Principle of super position**
  - The principle of super position states that for linear systems, the *individual responses* to several disturbances or driving functions *can be superposed on each other to obtain the total response* of the system.

- **Linear system:** A system is linear if it satisfies the principle of superposition (additivity and homogeneity). Newton's equations for dynamic equilibrium are linear. Example: Spring system.

- **Non-linear system:** Opposite of linear system. Its output is not directly proportional to input. Example: Systems with static or coulomb friction, backlash.

Force Analysis Techniques …

➢ Superposition

- Given a mechanism with known position, velocity, and acceleration conditions, *derive Newtons equations for dynamic equilibrium*. These equations are linear in the forces and therefore **superposition** principles can be applied
- Inertial and applied forces on each link can be considered individually and then superposed to determine their combined effect.
- This approach is good for building intuition and solving by hand.
- This approach can be very long
- Principle of super position has a limitation that it cannot be applied for non-linear systems.

Force Analysis Techniques ...

Superposition

- To find driving and bearing forces using superposition, break the problem into *n-1* parts. For example...
  - Find Tin’ = due to forces on link *n*
  - Find Tin” = due to forces on link *n-1*
  - Find Tin””” = due to forces on link 2
- Notes:
  - Notation: F14 = force of link 1 on link 4
  - Be careful with signs
  - F14=-F41,
  - But, remember to either switch the sign, or the direction of the vector, but not both
  - ***T=rxF***

Force Analysis Techniques …

Superposition: Procedure

- Break the problem into *n-1* parts, *n* = number of links.
- Start with part 1
  - Draw FBD’s of the extreme link
  - Include all forces **for Part 1 ONLY**
  - Look for 2FM’s to reduce unknowns
  - Solve unknown reaction forces as 3 eq’s, 3 uk’s (in general)
  - 3 equations are:

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum M_g = I_g \alpha$$

Force Analysis Techniques …

Superposition: Procedure (cont.)

- Move to the next link, transferring forces from the previous as, $F_{23}=-F_{32}$ (or, changing the directions on the vectors and keeping the magnitude)
- Continue to the driving link, solve for $_{Tin'}$

➢ Move to part II

- Repeat, solving $T_{in''}$

➢ Continue for all parts, 1 to *n*-1

➢ Find the total reactions as:

- $T_{in(tot)} = T_{in'}+T_{in''}+T_{in'''}+ \ldots$
- $F_{12}x(tot)= F_{12}x'+ F_{12}x''+\ldots$, etc.

➢ Note, consistent directions for all the reaction forces must be maintained in all parts.

Force Analysis Techniques …

- **Matrix Method**
  - Inertial and applied forces are considered at once.
  - The dynamic equations become coupled in the unknown forces and are solved using linear algebra techniques.
  - This approach is the best for computer application, therefore the method of choice for complex analyses.

Force Analysis Techniques …

- **Energy method (Virtual work)**:
  - An equation of conservation of energy is written that results in 1 scalar equation with 1 unknown (for a 1 DoF system)
  - Here, only forces that do work on the mechanism are considered.
  - This is the easiest method if only the input force is required.

Lesson 2 Revision Problems

1) Name the three methods commonly used in force analysis.
2) Explain the matrix method of force analysis. What are the strengths of the matrix method over the competing methods?
3) Discuss the main limitation(s) of the virtual work/energy method of force analysis.
4) State the principle of superposition. What are the main steps of the superposition method of force analysis?
5) Compare and contrast linear and non-linear systems. Give one example of each linear system and nonlinear system.

End…

Any Questions?

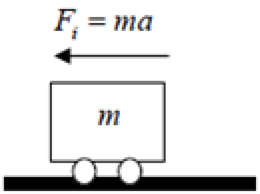
Lesson 3

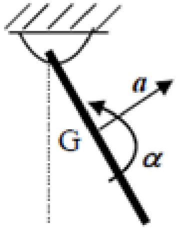
Static Force Analysis

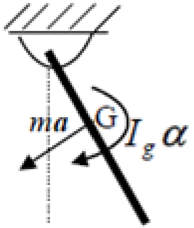
Static Force Analysis …

In static force analysis, of the forces acting on the machine members at a particular configuration:

- Each configuration is considered as system of rigid bodies in static equilibrium under the action of external forces.
- Load carried by each part of the machine during cycle of its operation is determined in this way by using static equilibrium equations.

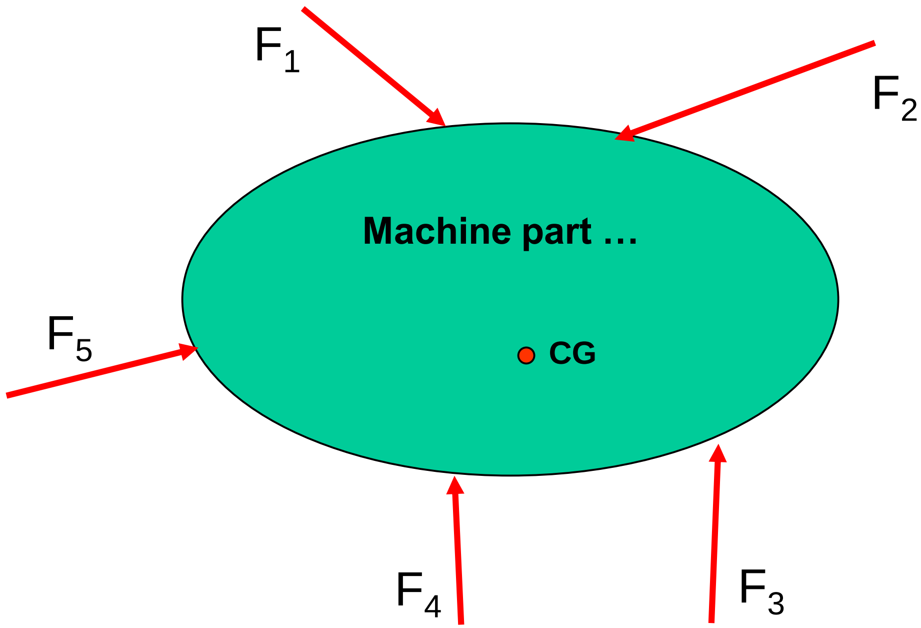






Static Force Analysis …

Condition for static equilibrium



(Multi-force member easiest method if only the input force is required)

Equations for static equilibrium

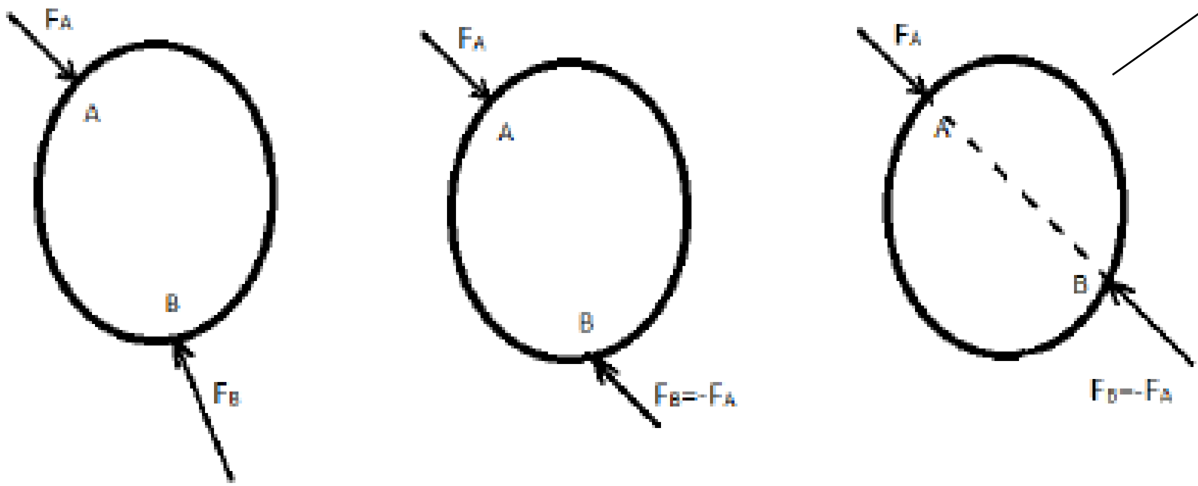
$$\sum F_i = 0$$

$$\sum M_i = 0$$

Static Force Analysis …

Equilibrium of two force and three force members

- Two force member in equilibrium



Not in equilibrium
$\Sigma F \neq 0, \ \Sigma M \neq 0$

Not in equilibrium
$\Sigma F = 0, \ \Sigma M \neq 0$

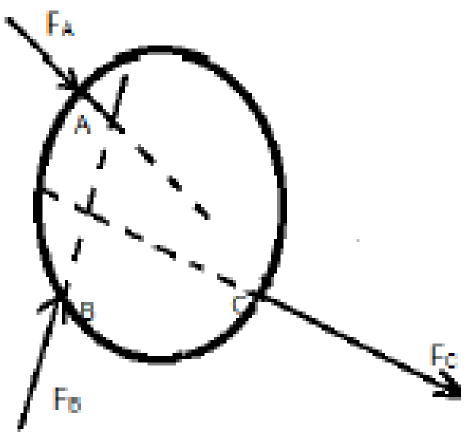
In equilibrium
$\Sigma F = 0, \ \Sigma M = 0$

A Two force member is a body that has 2 forces (only forces/no moments) acting on it in 2 locations).

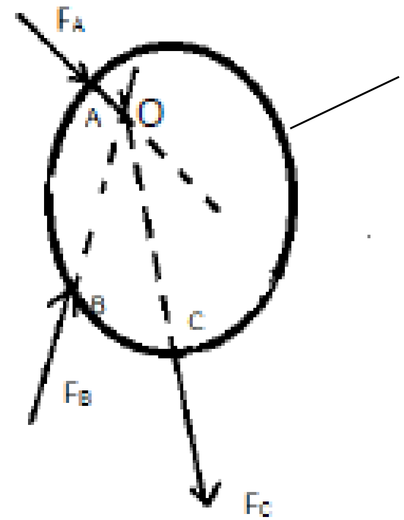
Condition of equilibrium, for any two-force member with no applied torque: The forces are equal, opposite and have the same line of action.

Static Force Analysis …

- Three force member in equilibrium



Not in equilibrium
$\Sigma M \neq 0$



In equilibrium
$\Sigma F=0, \ \Sigma M=0$

If three non parallel forces (only forces/no moments) act on a body in equilibrium, it is known as a three force member.

Condition of equilibrium, for any three-force member with **no applied torque:**
Forces should be coplanar

Static Force Analysis …

Free-Body Diagram (FBD) for a Link

- FBD: A diagram of a link with all forces and moments (external and internal) applied to it.
- Under the action of these forces (static and inertial) and moments, the link must be in equilibrium.

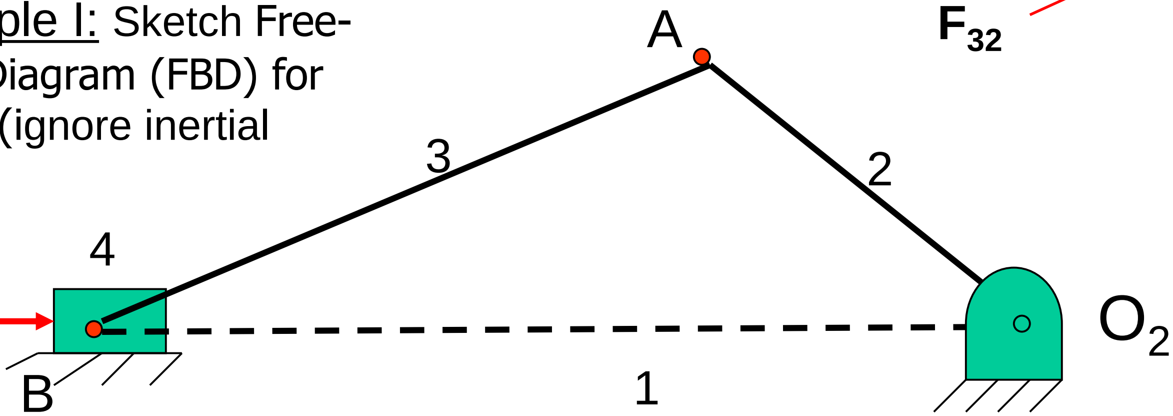
$$\sum_{i=1}^{n} F_i = 0 \qquad \sum_{j=1}^{m} M_j = 0$$

- These equations result in relationships between the known and unknown forces for a single link.

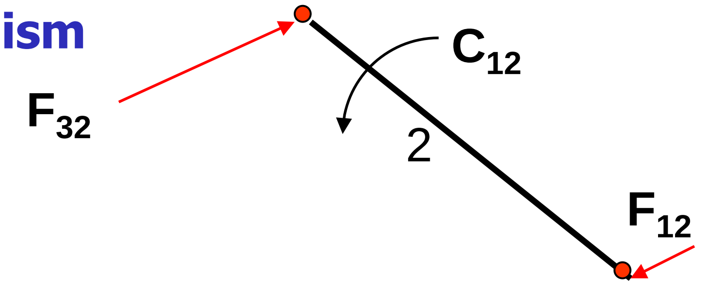
Static Force Analysis …

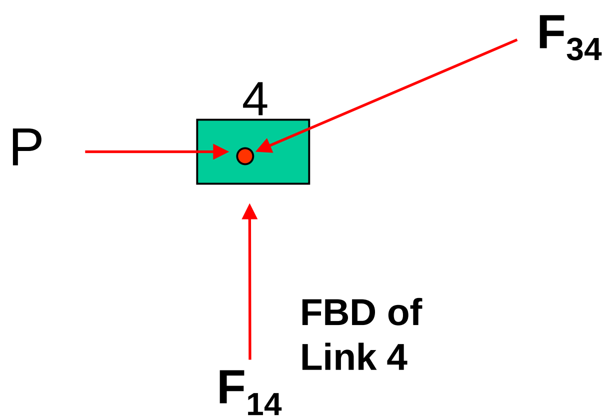
Static force analysis in slider-crank mechanism

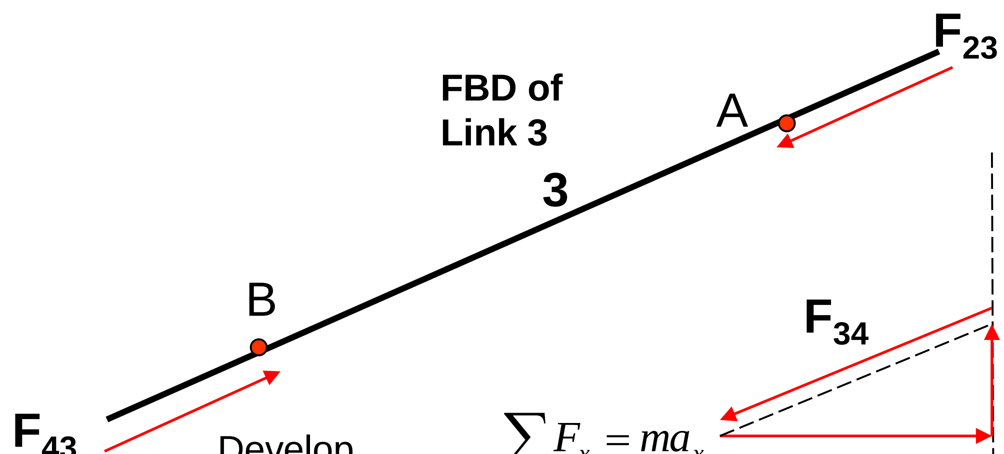
<u>Example I:</u> Sketch Free-Body Diagram (FBD) for a Link (ignore inertial forces)



FBD of Link 2





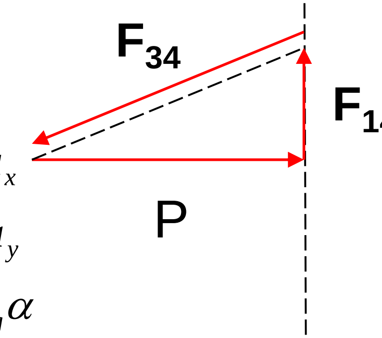


Develop equilibrium equations and find unknowns

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

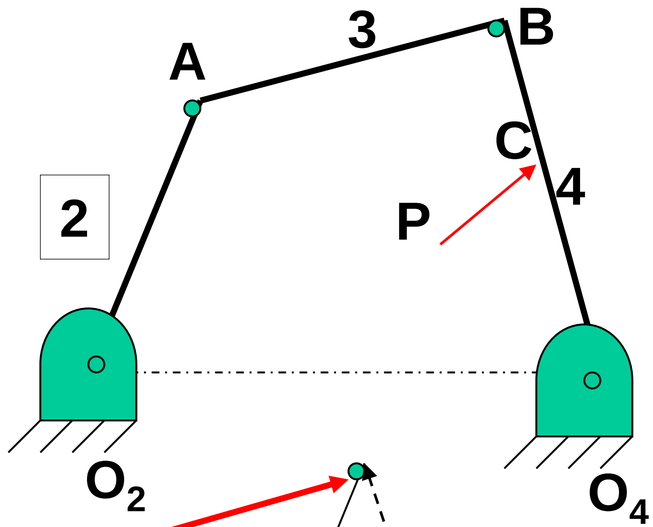
$$\sum M_g = I_g \alpha$$

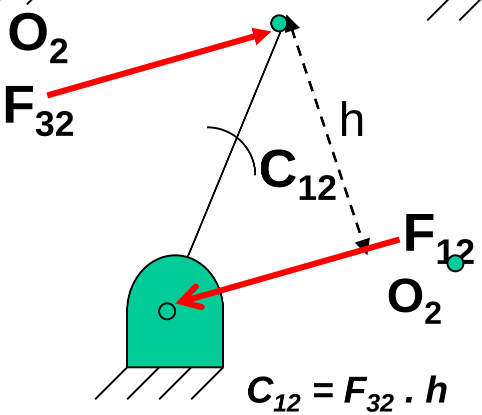


Static Force Analysis …

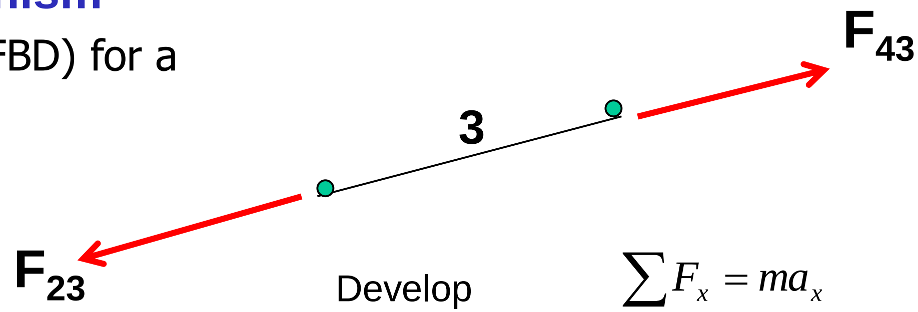
Static force analysis in 4-bar mechanism

Example II: Sketch Free-Body Diagram (FBD) for a Link (ignore inertial forces)



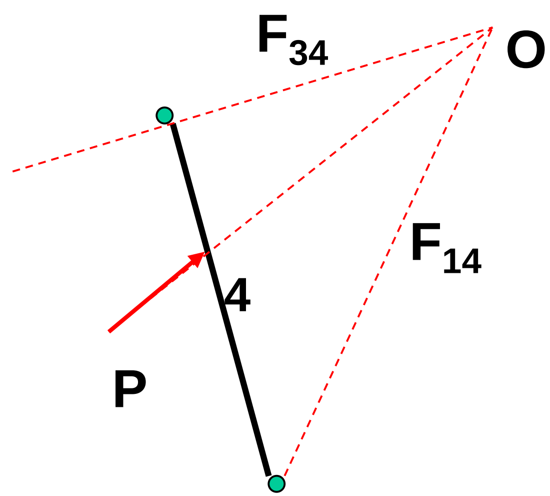


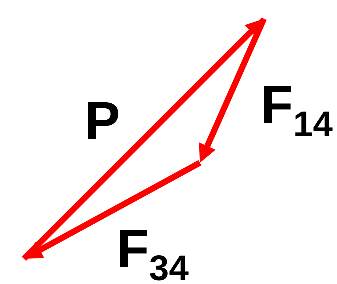
$C_{12} = F_{32} \cdot h$



Develop equilibrium equations and find unknowns

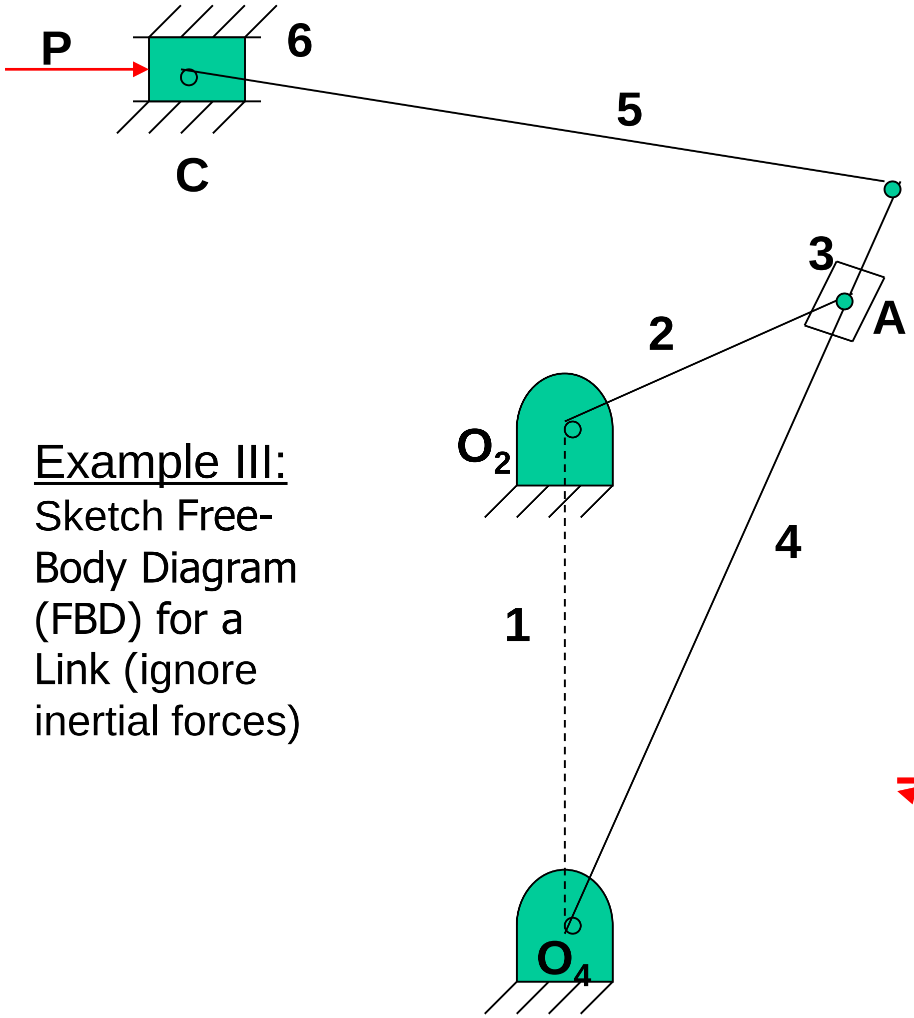
$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum M_g = I_g \alpha$$

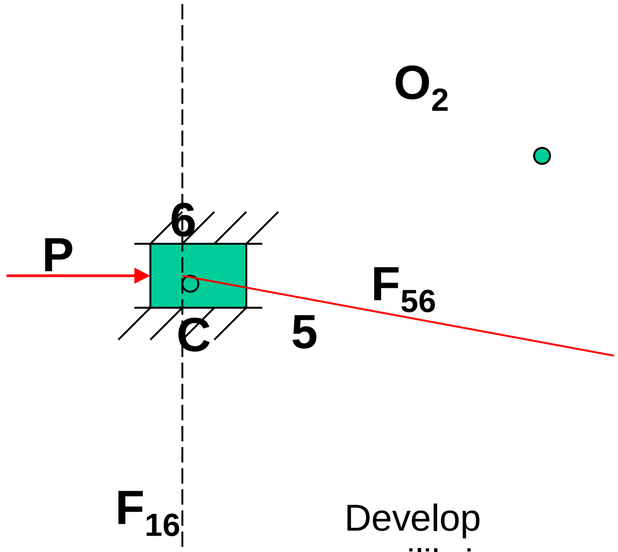




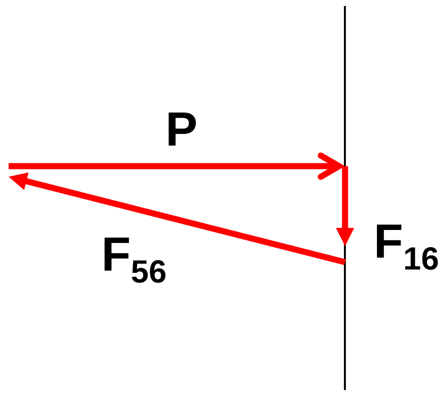
Static Force Analysis …

Static force analysis in 6-bar mechanism





Example III:
Sketch Free-Body Diagram (FBD) for a Link (ignore inertial forces)

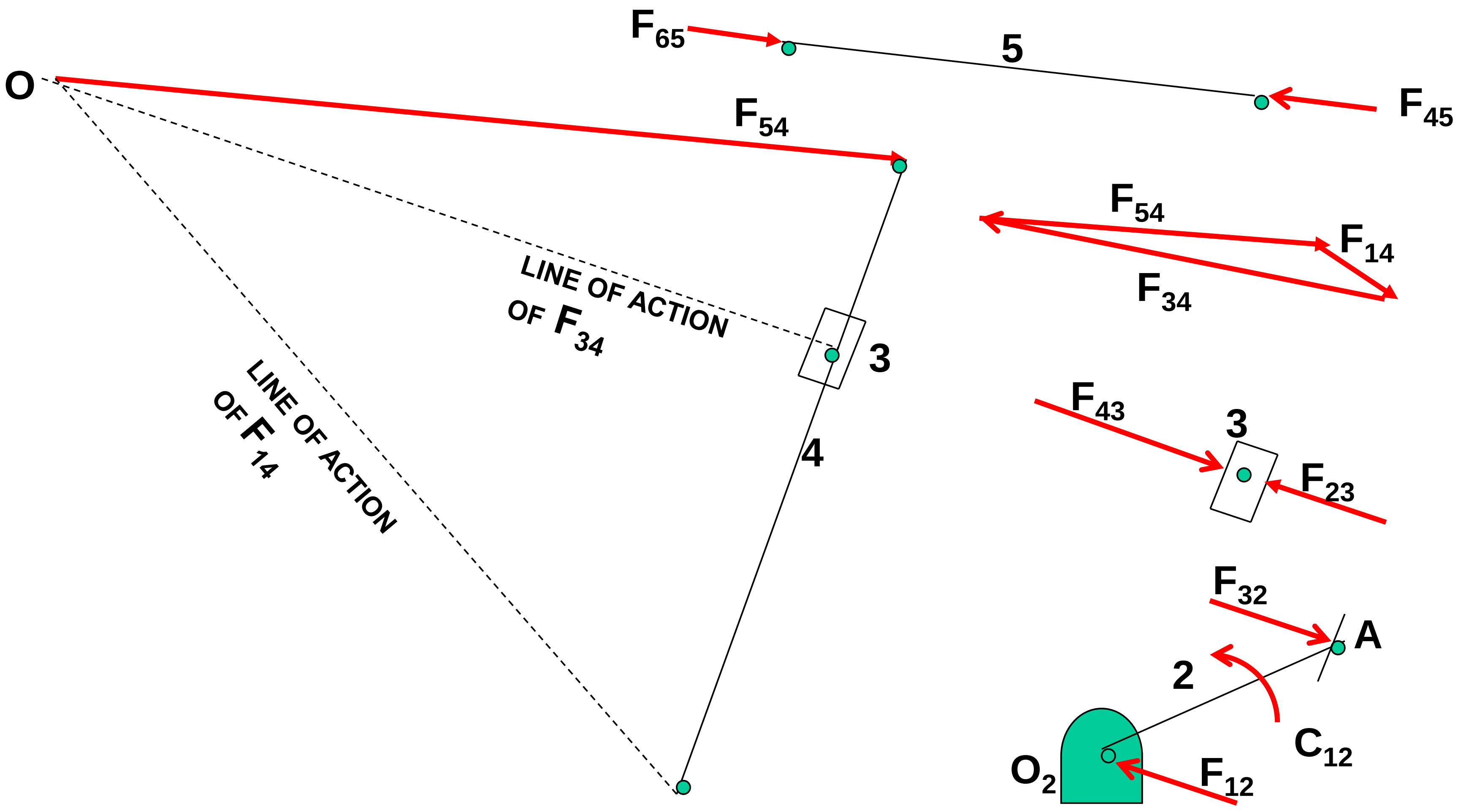


Develop equilibrium equations and find unknowns

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum M_g = I_g \alpha$$

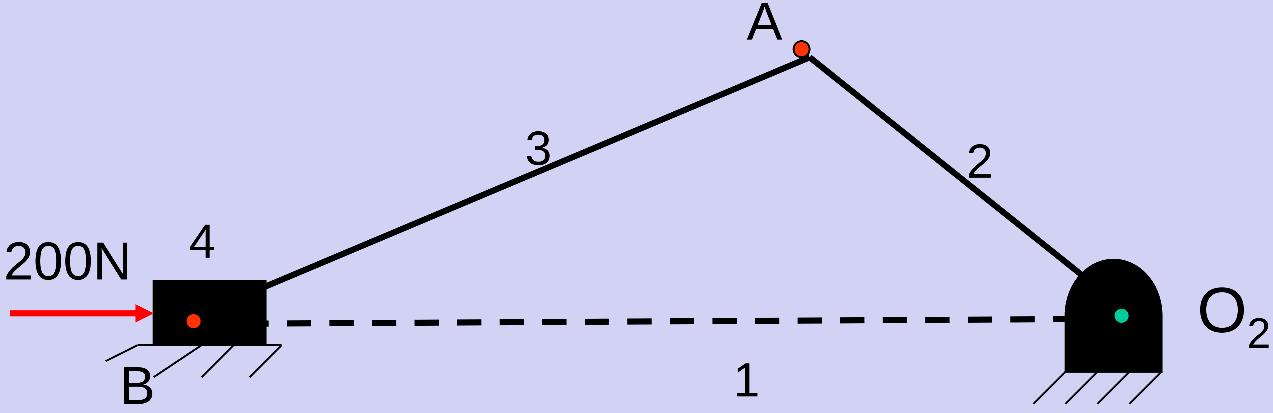
Static Force Analysis …

Static force analysis in 6-bar mechanism (continued)



Lesson 3 Revision Problems

1) Define the following terms: (a) two-force member, (b) three-force member, (c) free-body diagram.
2) Describe the static force analysis approach.
3) For the crank-slider mechanism shown below (a) sketch free-body diagram of the slider, (b) determine the normal force on the slider if the angle between the ground and the coupler is 20°.



End…

Any Questions?

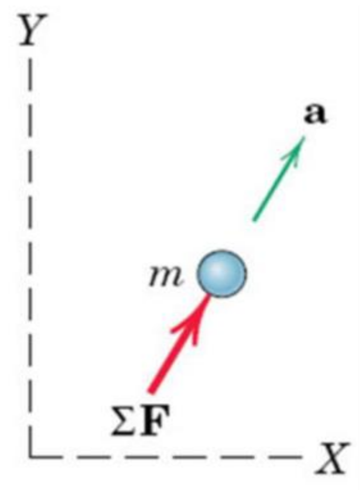
Lesson 4

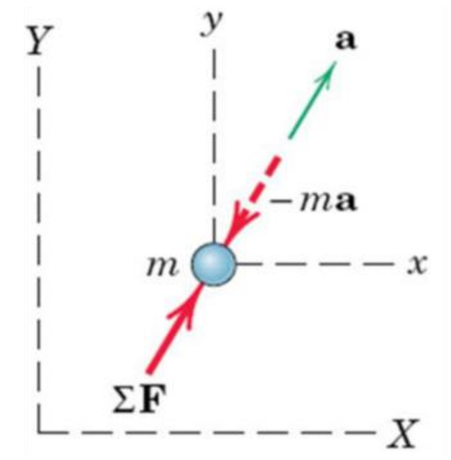
Dynamic Force Analysis

Dynamic Force Analysis …

Dynamic force analysis in mechanisms

- When inertia forces are considered in analysis of a mechanism, then the analysis is known to as dynamic force analysis.
- The inertia force is an *imaginary* force, which acts upon a rigid body.
- It is numerically equal to the accelerating force in *magnitude* and *Opposite in direction*.





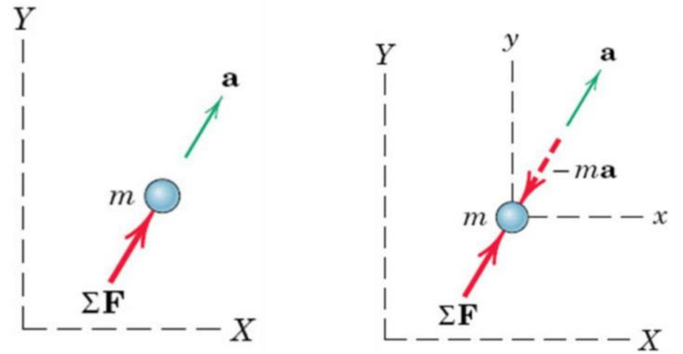
Dynamic Force Analysis …

- Mathematically,

  Inertia force = – Accelerating force = $-\,m.a$

  - Where, ***m*** = Mass of the body, and a = Linear acceleration of the center of gravity of the body.

- Similarly, the inertia torque is an imaginary torque; which when applied upon the rigid body, brings it in equilibrium position.
- It is equal to the accelerating couple in magnitude but ***opposite in direction*.**



- Inertia force - $m.a$
- Inertia torque - $I.\alpha$

Dynamic Force Analysis …

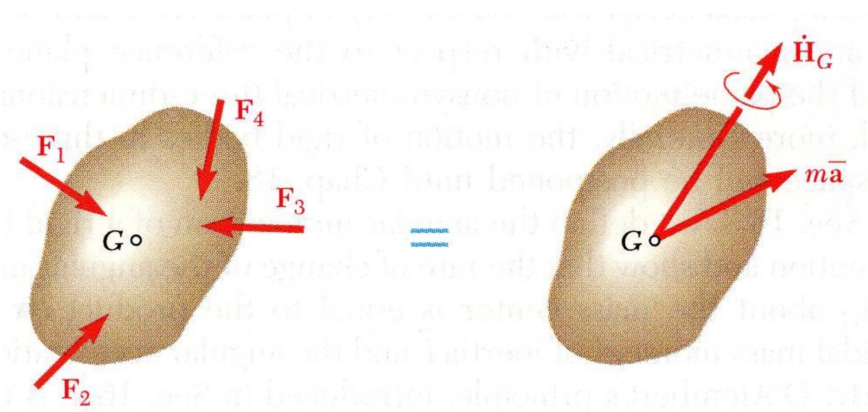
Dynamic equilibrium of mechanisms

D' Alemberts principle

- D'Alembert's principle states that the inertia forces and torques, and the external forces and torques acting on a body together result in statical equilibrium.
- In other words,
  - the vector sum of all external forces acting upon a system of rigid bodies is zero.
  - The vector sum of all external moments and inertia torques acting upon a system of rigid bodies is also separately zero.

Dynamic Force Analysis …

Equations of Motion for a Rigid Body



- Consider a rigid body acted upon by several external forces.
- For the motion of the mass center *G* of the body with respect to the Newtonian frame *Oxyz*,

$$\sum \vec{F} = m\vec{\bar{a}}$$

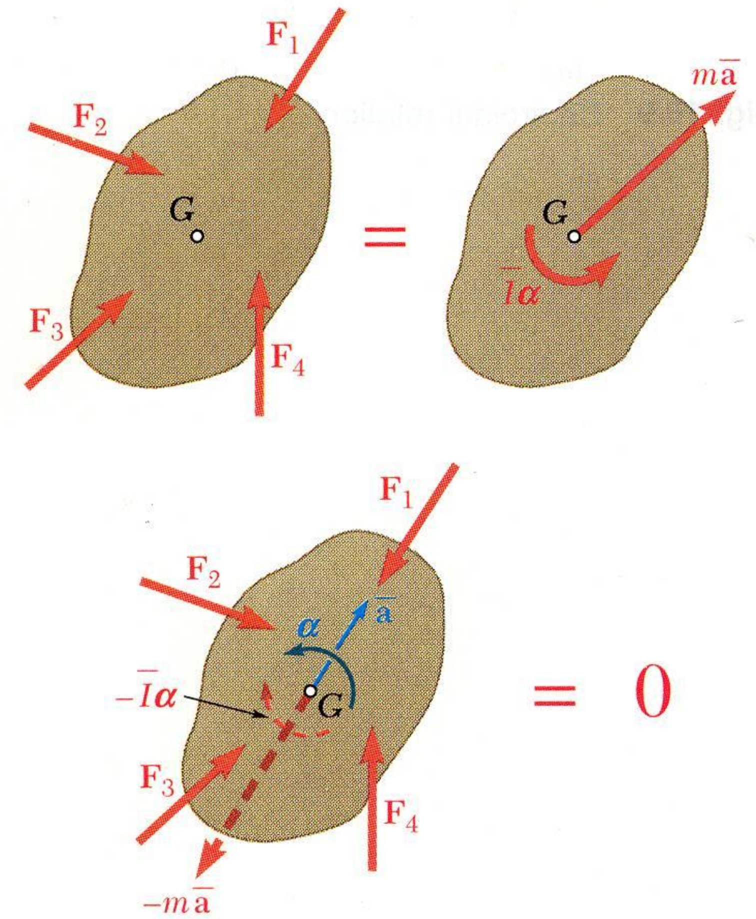
- For the motion of the body with respect to the centroidal frame *Gx′y′z′*,

$$\sum \vec{M}_G = I\alpha$$

- The general motion can be expressed by the sum of a translation and a centroidal rotation.

Dynamic Force Analysis …

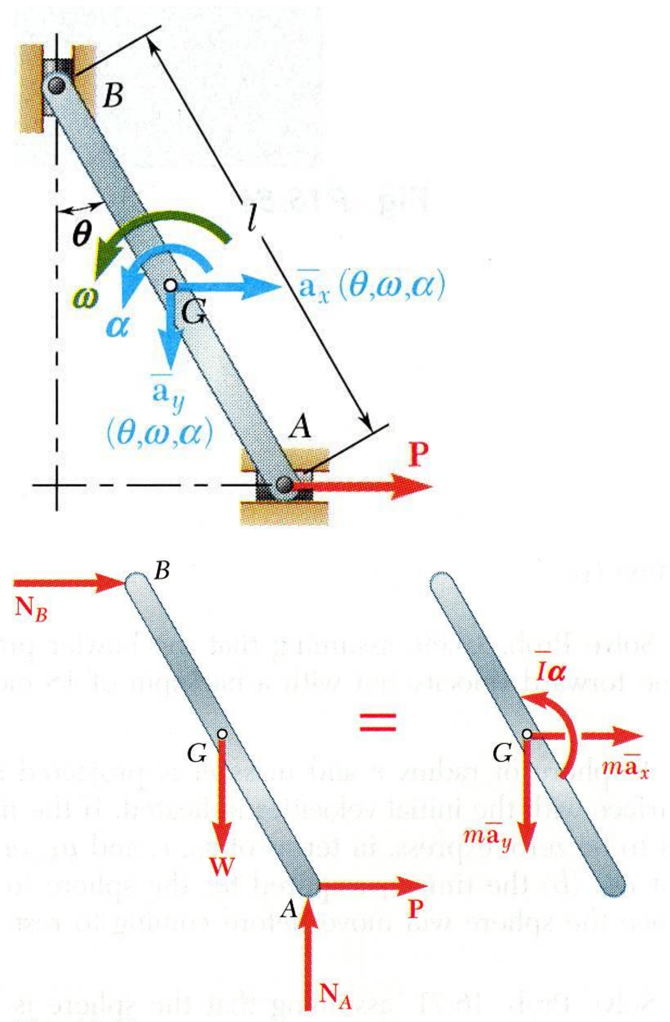
Solution of Problems Involving the Motion of a Rigid Body



- Forces are replaced by the sum of a translation and a centroidal rotational forces.
- Recall the D'Alembert's principle: the inertia forces and torques, and the external forces and torques acting on a body together result in statical equilibrium.
- Thus, the techniques for solving problems of static equilibrium may be applied to solve problems of plane motion.

Dynamic Force Analysis …

Solution of Problems Involving the Motion of a Rigid Body



- Given θ, ω, and α, find P, N_A, and N_B.
- Kinematic analysis yields $\bar{a}_x$ and $\bar{a}_y$.
- Application of D'Alembert's principle yields P, N_A, and N_B

Develop equilibrium equations and find unknowns

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum M_g = I_g\alpha$$

Lesson 4 Revision Problems

1) Write down mathematical expressions used to determine or solve the following: (a) inertia force, (b) inertia torque, (c) problems involving the motion of a rigid body.
2) Explain the dynamic force analysis process.
3) State the D'Alembert's principle.

End…

Any Questions?

Lesson 5

Dynamic Analysis of Reciprocating Engines

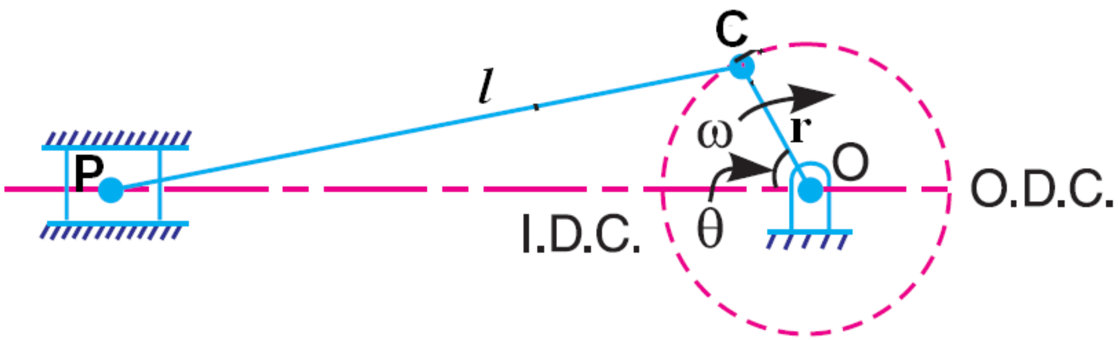
Dynamic Analysis of Reciprocating Engines ...

Force Analysis in IC Engine Mechanism

- The study of forces in IC engine mechanism are classified into two types as follows :
  - **Static force analysis**
  - **Dynamic force analysis**
- In **static force analysis**, we do not consider the effect of inertia forces arising <u>due to the mass of the connecting rod.</u>
- In **dynamic force analysis**, <u>we also consider</u> the effect of inertia forces caused due to the mass of connecting rod.
- The force analysis can be done both by analytical and graphical methods.

Dynamic Analysis of Reciprocating Engines …

Static Force Analysis of IC Engine Mechanism

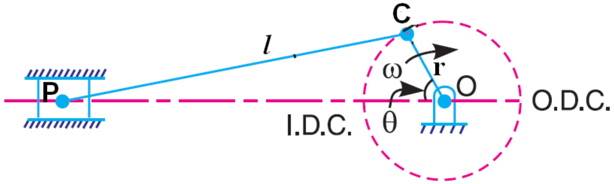


Slider crank mechanism.

- Consider an IC engine mechanism shown in Fig. in which the crank OC rotates at an angular speed of ω in clockwise direction.
- … at an instant it is inclined at angle θ from I.D.C.

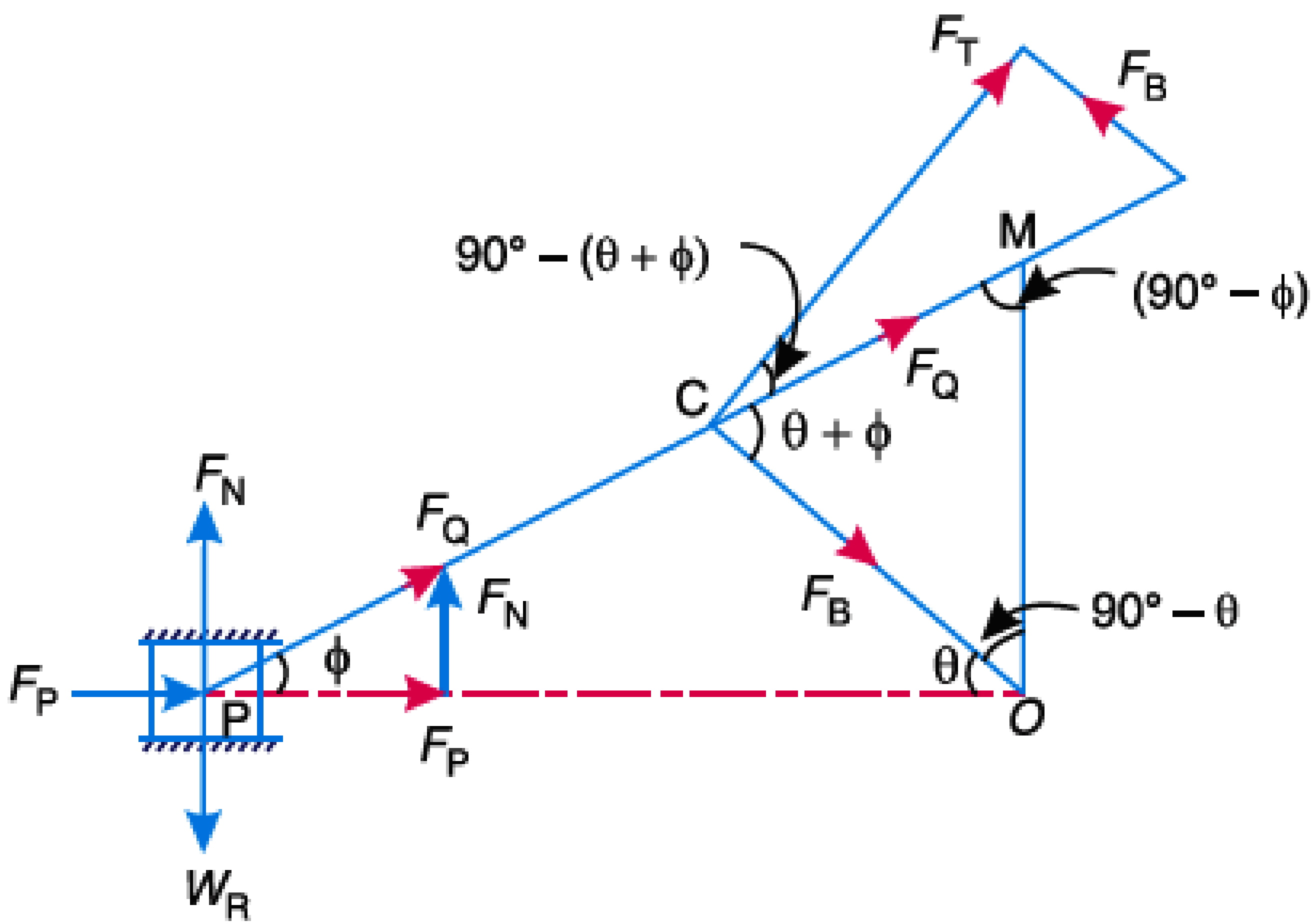
Dynamic Analysis of Reciprocating Engines …

- Let,
  - **r** = Radius of crank,
  - **l** = Length of connecting rod,
  - **n** = Obliquity ratio = l/r
  - $\mathbf{m_R}$ = Mass of the reciprocating parts, e.g. piston, crosshead pin or gudgeon pin etc., in kg,
  - $\mathbf{W_R}$ = Weight (in N) of the reciprocating parts $= m_R.g$
  - **m** = Mass of the connecting rod,
  - **θ** = Angle made by crank with I.D.C.,
  - **Φ** = Angle made by connecting rod with line of reciprocation (i.e. line OP) when line is inclined at angle θ



Slider crank mechanism.

Dynamic Analysis of Reciprocating Engines …



Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

Piston effort

- Is the net force acting on the piston or crosshead pin, along the line of stroke… denoted by F_P (in the Fig).
- The acceleration of the reciprocating parts,

$$a_R = a_P = \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

- Accelerating force or inertia force of the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

- Therefore, Piston effort, F_P = Net load on the piston ± Inertia force

$= F_L \pm F_I$ *...(Neglecting frictional resistance)*

$= F_L \pm F_I - R_F$ *...(Considering frictional resistance)*

where, R_F = *Frictional resistance.* The –ve sign is used when the piston is accelerated, and +ve sign is used when the piston is retarded.

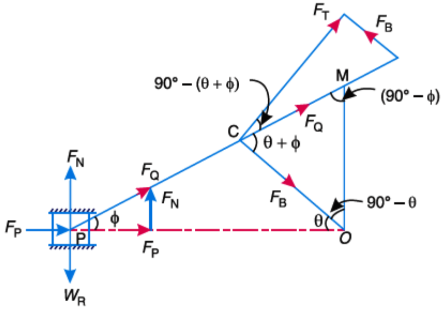
Dynamic Analysis of Reciprocating Engines …

- In a reciprocating steam engine, net load on the piston,

  $$F_L = p_1 A_1 - p_2 A_2 = p_1 A_1 - p_2 (A_1 - a)$$

  where,

  - p_1, A_1 = Pressure and cross-sectional area on the back end side of the piston,
  - p_2, A_2 = Pressure and cross-sectional area on the crank end side of the piston,
  - a = Cross-sectional area of the piston rod.



Forces on the reciprocating parts of an engine.

- **Notes :**
  - If 'p' is the net pressure of steam or gas on the piston and D is diameter of the piston, then in case of a vertical engine,

Net load on the piston, F_L = Pressure × Area = $p \times \frac{\pi}{4} \times D^2$

Piston effort, $F_P = F_L \mp F_I \pm W_R - R_F$

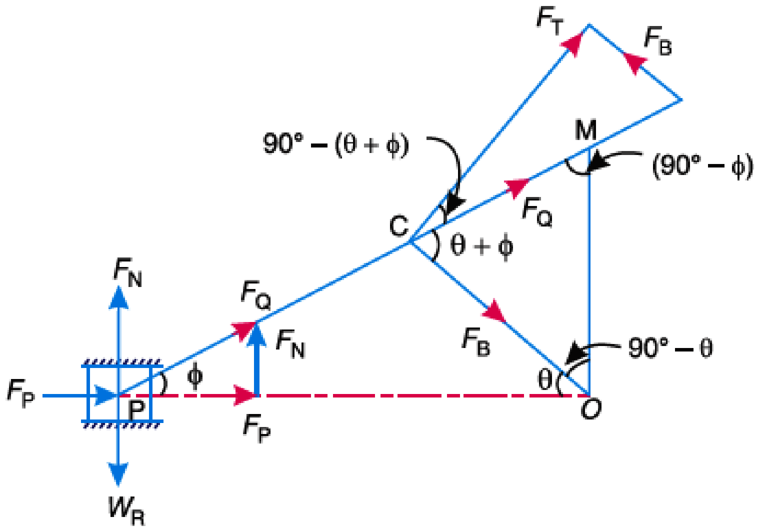
Dynamic Analysis of Reciprocating Engines …

- **Force acting along the connecting rod.**
  - It is denoted by F_Q in Fig. From the geometry of the figure, we find that

$$F_Q = \frac{F_P}{\cos\phi}$$

We know that $\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$

$$\therefore \qquad F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

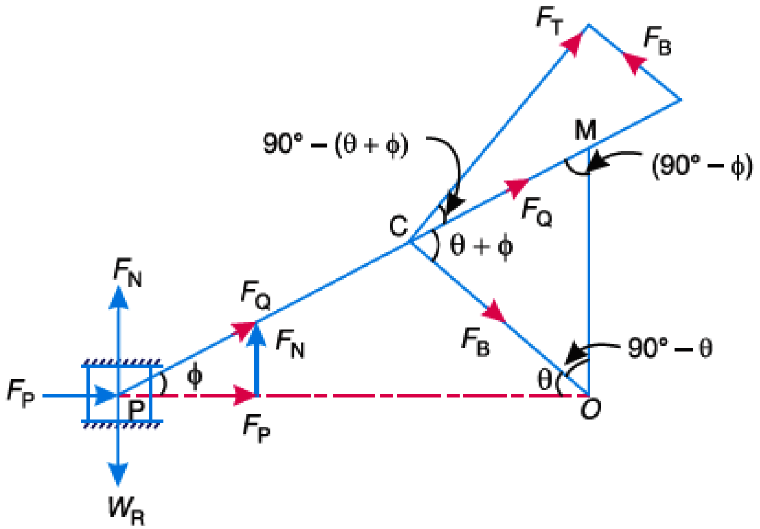


Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

- **Thrust on the sides of the cylinder walls or normal reaction on the guide bars.**

  It is denoted by F_N in Fig. From the figure, we find that



Forces on the reciprocating parts of an engine.

$$F_Q = \frac{F_P}{\cos\phi}$$

$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi} \times \sin\phi = F_P \tan\phi \qquad \ldots\left[\because F_Q = \frac{F_P}{\cos\phi}\right]$$

Dynamic Analysis of Reciprocating Engines …

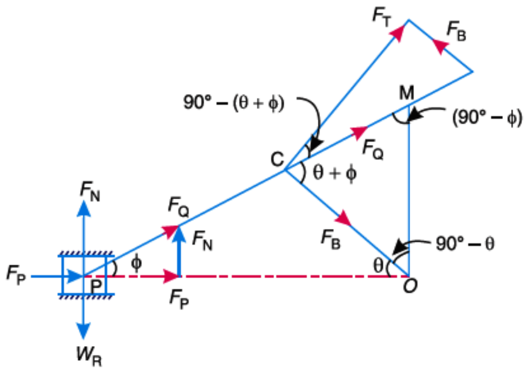
➢ Crank-pin effort and thrust on crank shaft bearings

- The force acting on the connecting rod F_Q may be resolved into two components, one perpendicular to the crank and the other along the crank.
- The component of F_Q perpendicular to the crank is known as crank-pin effort and it is denoted by F_T in Fig.
- The component of F_Q along the crank produces a thrust on the crank shaft bearings and it is denoted by F_B in Fig.
- Resolving F_O perpendicular to the crank,

$$F_T = F_Q \sin(\theta + \phi) = \frac{F_P}{\cos\phi} \times \sin(\theta + \phi)$$

- and resolving *FQ along the crank,*

$$F_B = F_Q \cos(\theta + \phi) = \frac{F_P}{\cos\phi} \times \cos(\theta + \phi)$$



Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

➢ Crank effort or turning moment or torque on the crank shaft

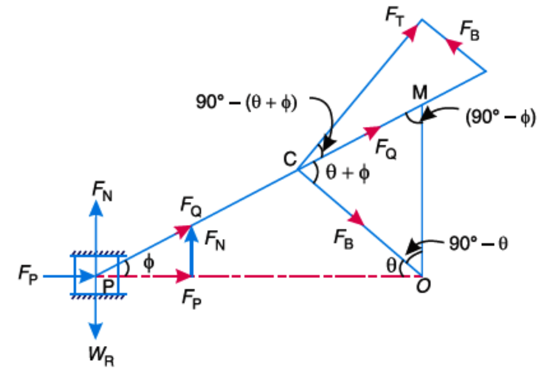
- The product of the crankpin effort (F_T) and the crank radius (r) is known as crank effort or turning moment or torque on the crank shaft.
- Mathematically, Crank effort,

$$T = F_T \times r = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r$$

$$= \frac{F_P (\sin\theta\cos\phi + \cos\theta\sin\phi)}{\cos\phi} \times r$$

$$= F_P\left(\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi}\right) \times r$$

$$= F_P(\sin\theta + \cos\theta\tan\phi) \times r \qquad \ldots(i)$$



Forces on the reciprocating parts of an engine.

We know that $l\sin\phi = r\sin\theta$

$$\sin\phi = \frac{r}{l}\sin\theta = \frac{\sin\theta}{n} \qquad \ldots\left(\because n = \frac{l}{r}\right)$$

and $$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

Dynamic Analysis of Reciprocating Engines ...

$$\therefore \quad \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{n} \times \frac{n}{\sqrt{n^2 - \sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Substituting the value of $\tan\phi$ in equation *(i)*, we have crank effort,

$$T = F_P\left(\sin\theta + \frac{\cos\theta\sin\theta}{\sqrt{n^2 - \sin^2\theta}}\right) \times r$$

$$= F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \quad \ldots(ii)$$

$$\ldots(\because 2\cos\theta\sin\theta = \sin 2\theta)$$

Note: Since $\sin^2\theta$ is very small as compared to n^2 therefore neglecting $\sin^2\theta$, we have,

Crank effort, $$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) = F_P \times OM$$

We have seen in Art. 15.8, that

$$OM = r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

Therefore, it is convenient to find OM instead of solving the large expression.

Lesson 5 Revision Problems

1) How is the study of forces and torque in IC engine mechanism classified?
2) What is piston effort? Explain how to determine piston effort.
3) How is net load on the piston in a reciprocating steam engine determined?
4) Explain how is thrust on the sides of the cylinder walls (or normal reaction on the guide bars) determined?
5) What is crank effort? Explain how to determine crank effort?
6) Use a suitable sketch to explain how to determine crank-pin effort and thrust on crank shaft bearings
7) Use a suitable sketch to show how to determine force acting along the connecting rod.

End…

Any Questions?

Lesson 6

Crank Shaft Torque

Crank Shaft Torque ...

Turning Moment ...

- We know that the turning moment on the crankshaft is

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

where F_P = Piston effort,
r = Radius of crank,
n = Ratio of the connecting rod length and radius of crank, and
θ = Angle turned by the crank from inner dead centre.

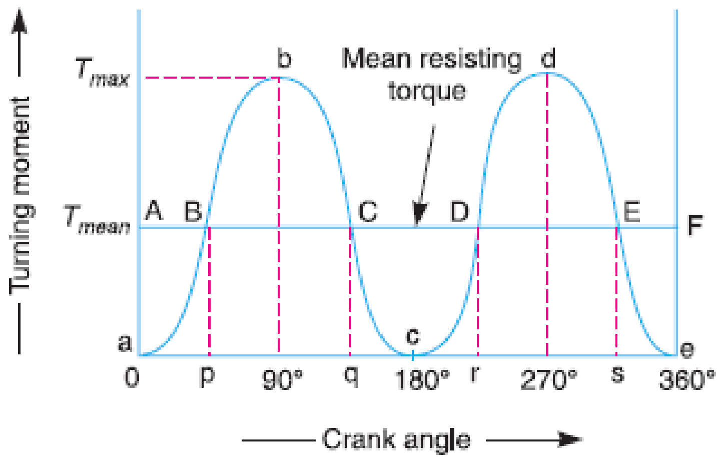
- From the above expression, we see that the turning moment (*T) is zero*, when the crank angle (θ) is zero.
- It is maximum when the crank angle is 90°, and
- It is again zero when crank angle is 180°.

Crank Shaft Torque …

Turning Moment Diagram …

- The turning moment diagram (also known as **crank effort diagram) is the graphical representation of the turning** moment or crank-effort for various positions of the crank.

- It is plotted on Cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.



Turning moment diagram for a single cylinder, double acting steam engine.

- The work done is the product of the turning moment and the angle turned
- Therefore the area of the turning moment diagram represents the work done per revolution.

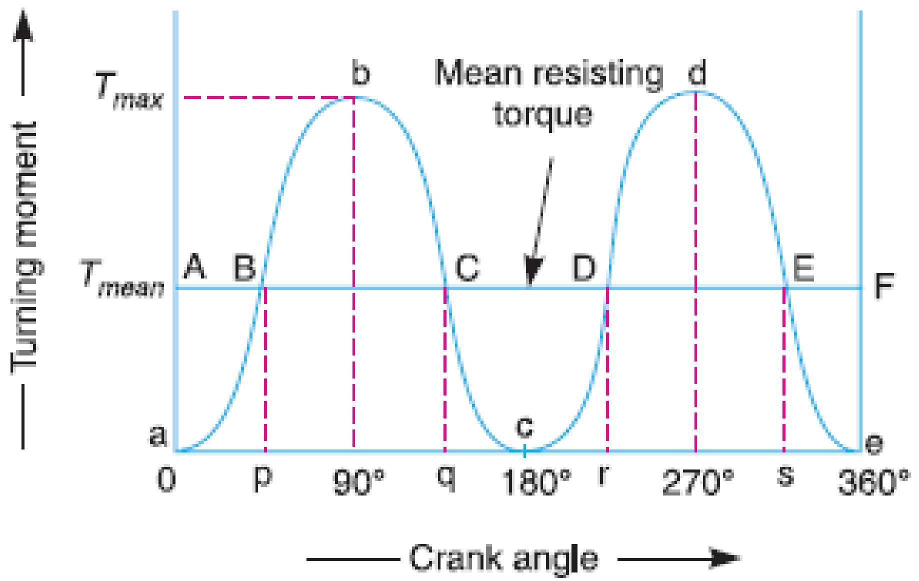
Crank Shaft Torque …

Turning Moment Diagram …

- **Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine**

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

The variations of energy above and below the mean torque line are called ***fluctuations of energy. The areas BbC, CcD, DdE, etc. represent fluctuations of energy.***



Turning moment diagram for a single cylinder, double acting steam engine.

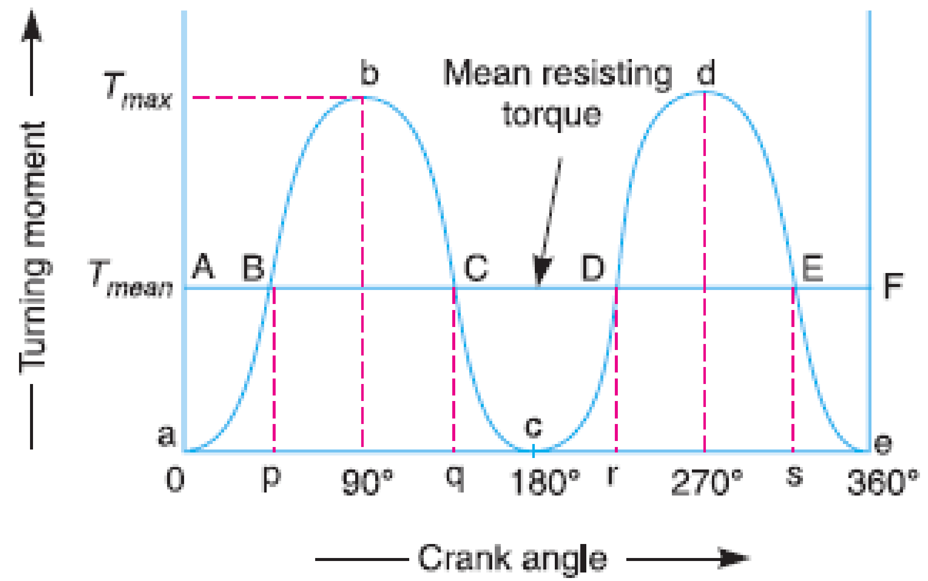
Crank Shaft Torque …

Turning Moment Diagram …

- **Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine**

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

The difference between the maximum and the minimum energies is known as maximum fluctuation of energy.

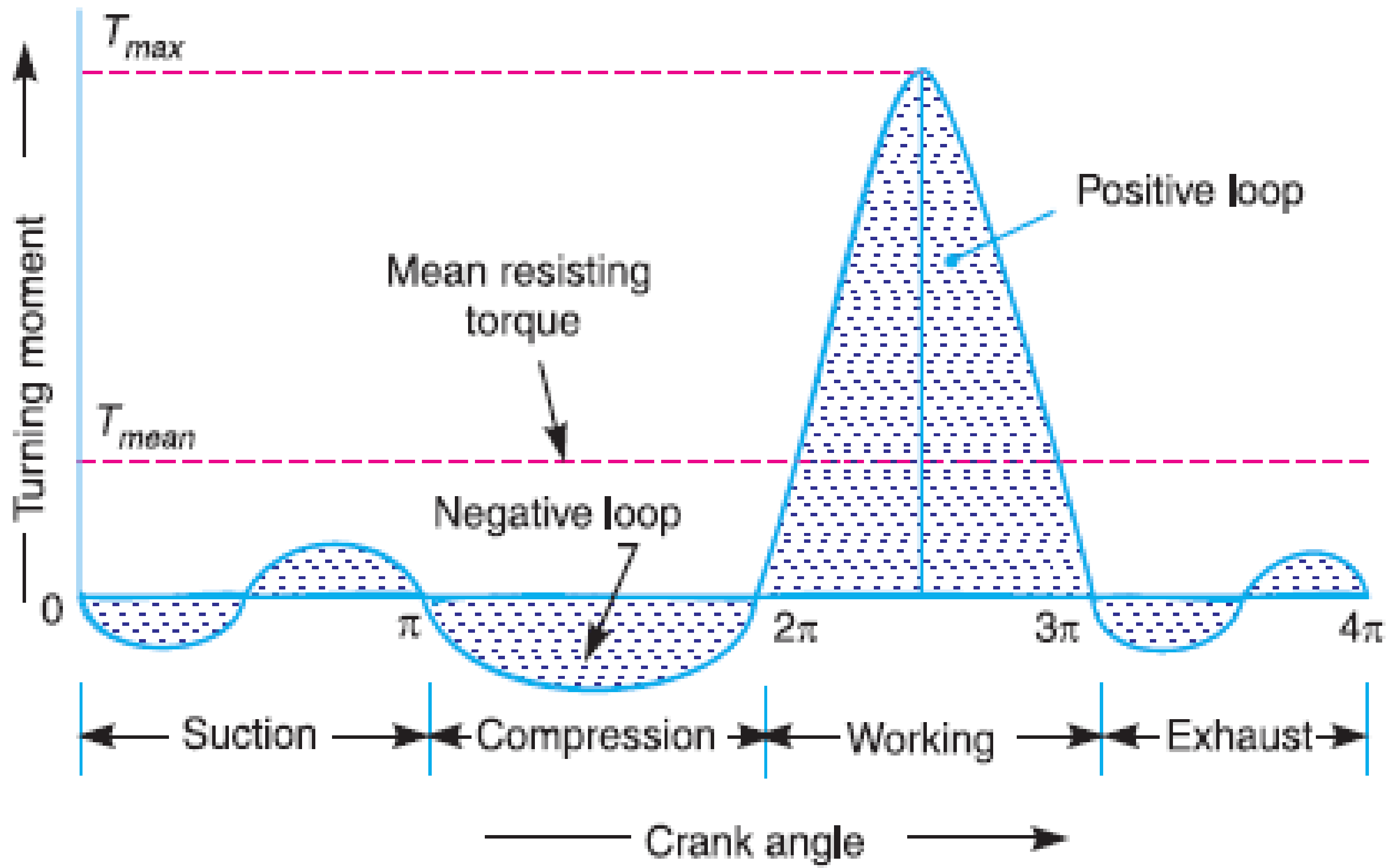


Turning moment diagram for a single cylinder, double acting steam engine.

Crank Shaft Torque …

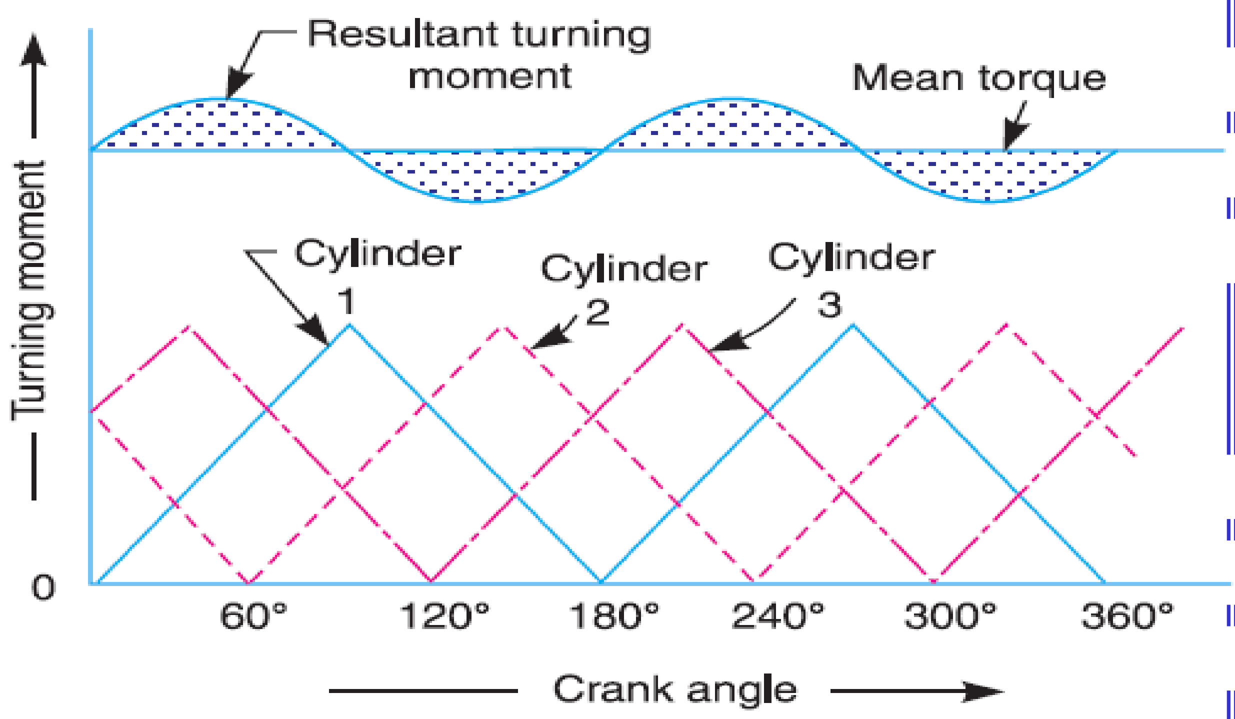
Turning Moment Diagram …

- Turning Moment Diagram for a Four Stock Cycle Internal Combustion Engine



Crank Shaft Torque …

Turning Moment Diagram …

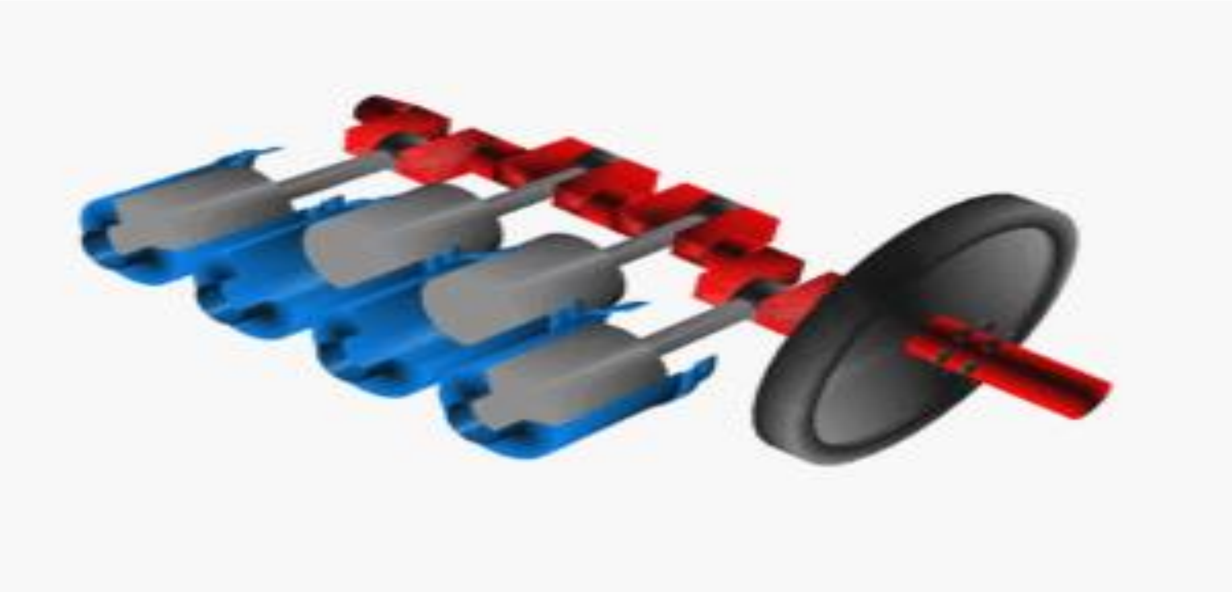


(Turning Moment Diagram for a Multi cylinder Engine)

- The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders.
- The cranks, in case of three cylinders, are usually placed at 120° to each other.

Crank Shaft Torque ...

Flywheel ...



- A rotating mechanical device that is used to store rotational energy during power stroke and delivers during idle strokes.

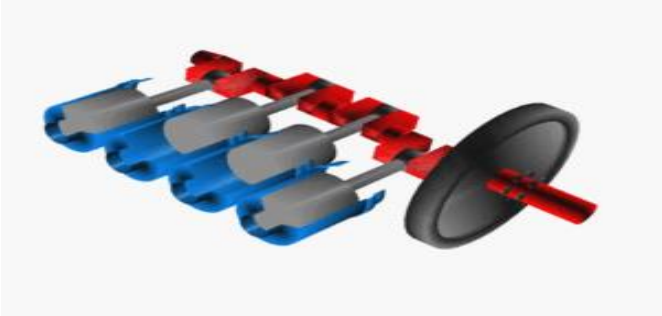
- The flywheel's position is between the engine and clutch patch to the starter.

- In true terms, a stroke is the upward or downward movement of the piston in the cylinder from TDC to BDC once (1)

Crank Shaft Torque …

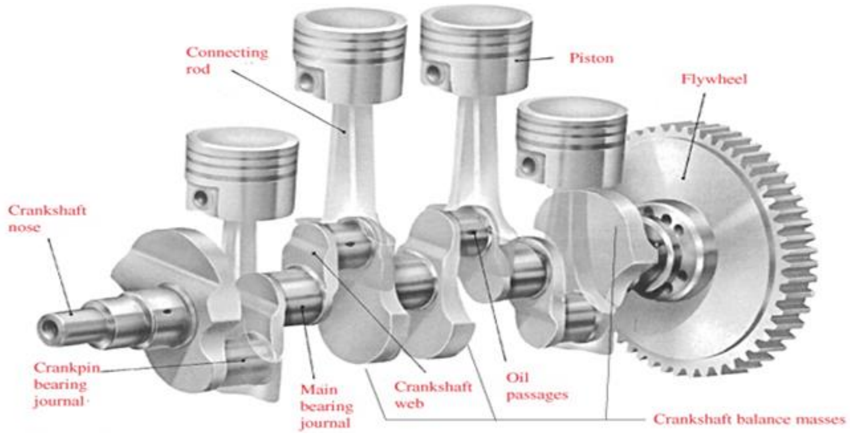
Flywheel …

- A flywheel serves as a reservoir which stores energy during the period where the supply of energy is more than the requirement.
- Conversely, a flywheel releases stored energy by applying torque to a mechanical load, thereby decreasing the flywheel's rotational speed.
- When the flywheel absorbs energy, its speed increases and when it releases, the speed decreases.
- Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.



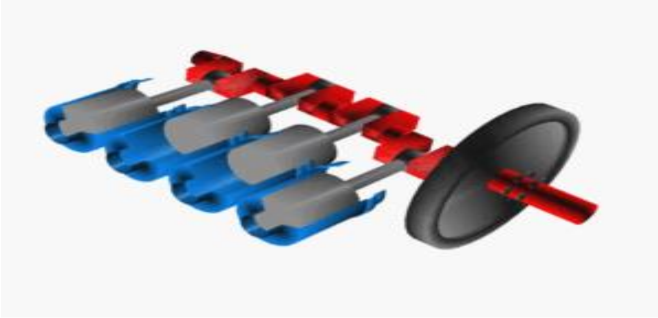
Crank Shaft Torque …

Flywheel …



A flywheel is used to maintain constant angular velocity of the crankshaft in a reciprocating engine. In this case, the flywheel—which is mounted on the crankshaft—stores energy when torque is exerted on it by a firing piston and it releases energy when no piston is exerting torque on it.

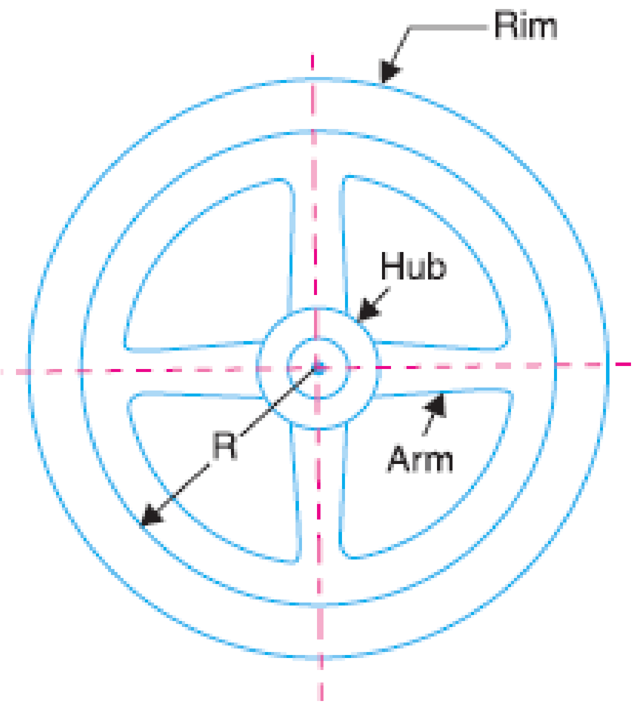
- In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.



Crank Shaft Torque …

Flywheel …

Mean speeds



N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

Where: N_1 and N_2 are maximum and minimum speeds in rpm; and ω_1 and ω_2 are maximum and minimum speeds in rad./sec.

Crank Shaft Torque …

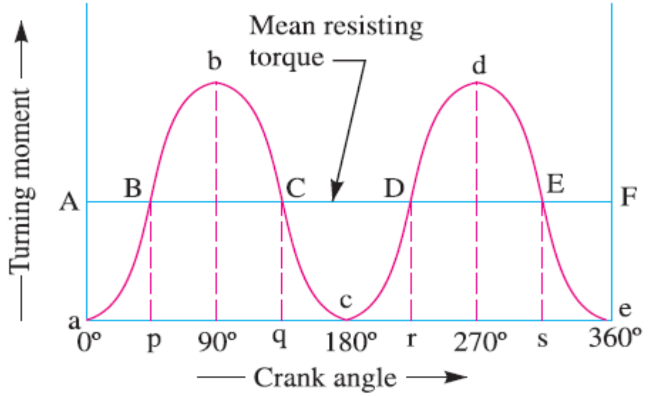
Flywheel …

Energy Stored in a Flywheel

- Rotational Kinetic Energy, $E = \frac{1}{2} I\omega^2$
  where,
  - I - moment of inertia of the flywheel (ability of an object to resist changes in its rotational velocity)
  - ω - rotational velocity (Rad / sec)

- The moment of inertia, $I = kMr^2$
  where,
  - M - mass of the flywheel
  - r - radius of flywheel
  - k - inertial constant.

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation.



Lesson 6 Revision Problems

1) What is turning moment? Write down the expression used to determine turning moment on a crank shaft.

2) What is the turning moment diagram (also known as crank effort diagram)?

3) Sketch turning moment diagrams of a: (a) three-cylinder engine, (b) four stroke IC engine.

4) Explain how turning moment diagram is used to determine work done.

5) What is flywheel? Show how the energy stored on a flywheel is determined.

End…

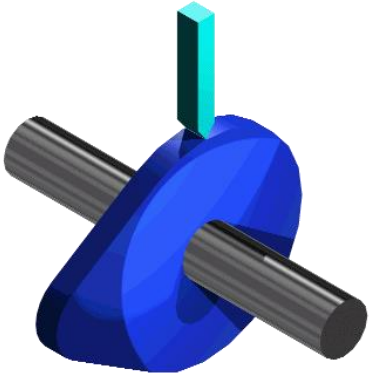
Any Questions?

Lesson 7

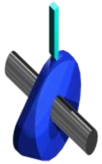
Cam Dynamics

Cam Dynamics …
(Terminology…)

- **Displacement (s)** – is the position of the follower from a specific zero or rest position in relation to time or the rotary angle of the cam.
- **Velocity (v)** – is the speed with which the cam moves the follower.
- **Acceleration (a)** – is the rate of change of the follower's velocity.
- **Jerk (j)** – is the rate of change of the follower's acceleration.



Cam Dynamics …

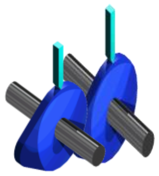


Follower motion schemes:

- Uniform motion (constant velocity)
- Simple harmonic motion
- Uniform acceleration and retardation/parabolic motion
- Cycloidal motion
- General polynomial motion

- Several different standard functions can be used to describe motions (i.e., displacement) of followers and used as the basis for constructing displacement diagrams.
- These displacement profiles/diagrams ultimately determine the shape of the cam.
- Also used in kinematic and dynamics analyses

Cam Dynamics …

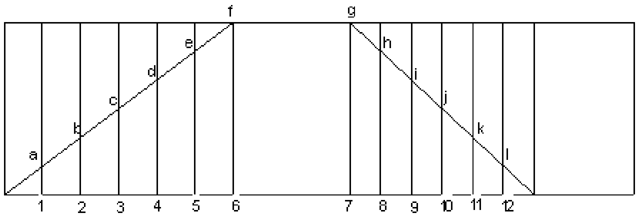


a) Uniform motion (constant velocity)

- The simplest follower motion …
- Motion is characterized with a straight-line displacement function/diagram because velocity is uniform.

$$y = H_0 + \frac{H_i \theta_i}{\beta_i} = H_0 + \frac{H_i t_i}{T_i}$$

- β is cam the rotation needed to achieve displacement *H. y—the rise portion of motion &* θ *are variables)*



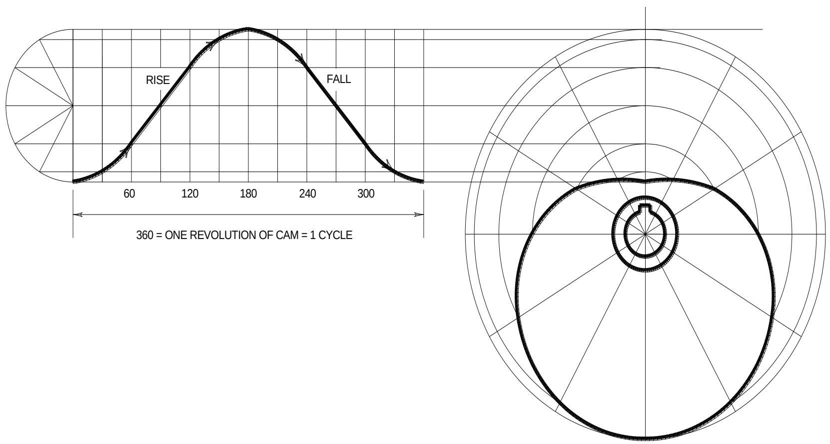
	Rise	Fall
Displacement:	$\Delta R_i = H_0 + \frac{H_i t_i}{T_i} = H_0 + \frac{H_i \phi_i}{\beta_i}$	$\Delta R_j = H_F + H_j\left(1 - \frac{t_j}{T_j}\right) = H_F + H_j\left(1 - \frac{\phi_j}{\beta_j}\right)$
Velocity:	$v_i = \frac{H_i}{T_i} = \frac{H_i \omega}{\beta_i}$	$v_j = \frac{-H_j}{T_j} = \frac{-H_j \omega}{\beta_j}$
Acceleration:	$a = 0$ (∞ at transitions)	$a = 0$ (∞ at transitions)

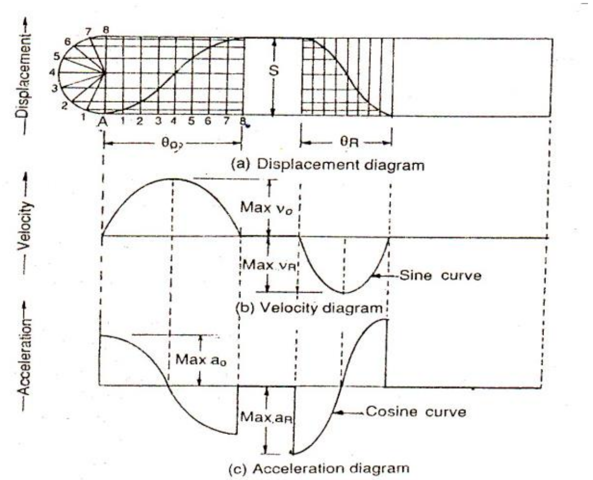
(displacement, velocity, and acceleration functions: - constant velocity rise and fall)

Cam Dynamics …

b) Simple harmonic motion

- Harmonic motion follower is derived from trigonometric functions, thus exhibiting very smooth motion curves.
- In a physical sense, it is the projection motion of a point on a rotating disk projected to a straight line.





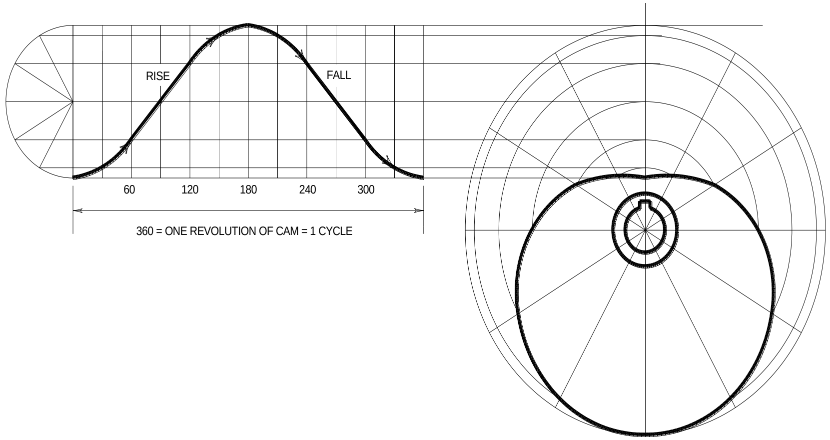
Cam Dynamics …

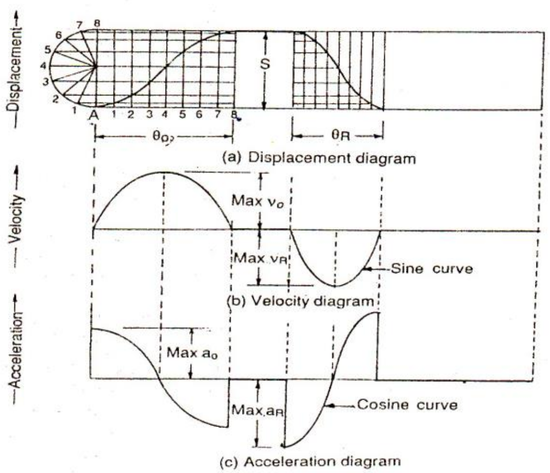
b) Simple harmonic motion

- The follower moves with a simple harmonic motion.
- Therefore velocity diagram is a sine curve and the acceleration diagram is a cosine curve.
- The "rise" portion of motion is defined by the following general function:

$$y = \frac{H_i}{2}\left(1 - \cos\frac{\pi\theta}{\beta_i}\right) = \frac{H_i}{2}\left(1 - \cos\frac{\pi t_i}{T_i}\right)$$

  - β is cam the rotation needed to achieve displacement *H. - (y & θ are variables)*





Cam Dynamics …

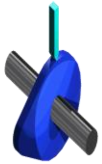
b) Simple harmonic motion

- Cam Follower Kinematics for Harmonic Motion: displacement, velocity, and acceleration functions

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + \frac{H_i}{2}\left[1 - \cos\left(\frac{\pi t_i}{T_i}\right)\right]$ $= H_0 + \frac{H_i}{2}\left[1 - \cos\left(\frac{\pi \phi_i}{\beta_i}\right)\right]$	$\Delta R_j = H_F + \frac{H_j}{2}\left[1 + \cos\left(\frac{\pi t_j}{T_j}\right)\right]$ $= H_F + \frac{H_j}{2}\left[1 - \cos\left(\frac{\pi \phi_j}{\beta_j}\right)\right]$
Velocity:	$v_i = \frac{\pi H_i}{2T_i}\left[\sin\left(\frac{\pi t_i}{T_i}\right)\right]$ $= \frac{\pi H_i \omega}{2\beta_i}\left[\sin\left(\frac{\pi \phi_i}{\beta_i}\right)\right]$	$v_j = \frac{-\pi H_j}{2T_j}\left[\sin\left(\frac{\pi t_j}{T_j}\right)\right]$ $= \frac{-\pi H_j \omega}{2\beta_j}\left[\sin\left(\frac{\pi \phi_j}{\beta_j}\right)\right]$
Acceleration:	$a_i = \frac{\pi^2 H_i}{2T_i^2}\left[\cos\left(\frac{\pi t_i}{T_i}\right)\right]$ $= \frac{\pi^2 H_i \omega^2}{2\beta_i^2}\left[\cos\left(\frac{\pi \phi_i}{\beta_i}\right)\right]$	$a_j = \frac{-\pi^2 H_i}{2T_j^2}\left[\cos\left(\frac{\pi t_j}{T_j}\right)\right]$ $= \frac{-\pi^2 H_j \omega^2}{2\beta_j^2}\left[\cos\left(\frac{\pi \phi_j}{\beta_j}\right)\right]$

Harmonic Rise **Harmonic Fall**

Cam Dynamics …



c) Uniform acceleration and retardation/parabolic

- Constant acceleration motion – general displacement, velocity, and acceleration functions

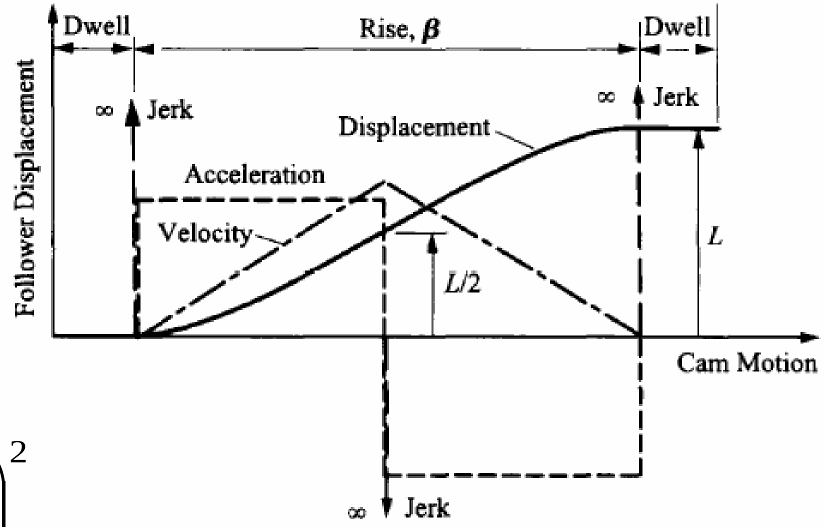
	Rise	Fall
For $0 < t < 0.5\,T\;(0 < \phi < 0.5\,\beta)$:		
Displacement:	$\Delta R_i = H_0 + 2H_i\left(\frac{t_i}{T_i}\right)^2 = H_0 + 2H_i\left(\frac{\phi_i}{\beta_i}\right)^2$	$\Delta R_i = H_F + H_j - 2H_j\left(\frac{t_j}{T_j}\right)^2 = H_F + H_j - 2H_j\left(\frac{\phi_j}{\beta_j}\right)^2$
Velocity:	$v_i = \frac{4H_i t_i}{T_i^2} = \frac{4H_i \omega \phi_i}{\beta_i^2}$	$v_j = \frac{-4H_j t_j}{T_j^2} = \frac{-4H_j \omega \phi_j}{\beta_j^2}$
Acceleration:	$a_i = \frac{4H_i}{T_i^2} = \frac{4H_i \omega^2}{\beta_i^2}$	$a_j = \frac{-4H_j}{T_j^2} = \frac{-4H_j \omega^2}{\beta_j^2}$
For $0.5\,T < t < T\;(0.5\,\beta < \phi < \beta)$:		
Displacement:	$\Delta R_i = H_0 + H_i - 2H_i\left(1 - \frac{t_i}{T_i}\right)^2 = H_0 + H_i + 2H_i\left(1 - \frac{\phi_i}{\beta_i}\right)^2$	$\Delta R_j = H_F + 2H_j\left(1 - \frac{t_j}{T_j}\right)^2 = H_F + 2H_j\left(1 - \frac{\phi_j}{\beta_j}\right)^2$
Velocity:	$v_i = \frac{4H_i}{T_i}\left(1 - \frac{t_i}{T_i}\right) = \frac{4H_i \omega}{\beta_i}\left(1 - \frac{\phi_i}{\beta_i}\right)$	$v_i = \frac{-4H_j}{T_j}\left(1 - \frac{t_j}{T_j}\right) = \frac{-4H_j \omega}{\beta_j}\left(1 - \frac{\phi_j}{\beta_j}\right)$
Acceleration:	$a_i = \frac{-4H_i}{T_i^2} = \frac{-4H_i \omega^2}{\beta_i^2}$	$a_j = \frac{4H_j}{T_j^2} = \frac{4H_j \omega^2}{\beta_j^2}$

(the dynamic characteristics of a constant acceleration rise)

Cam Dynamics …

c) Uniform acceleration and retardation/parabolic

- A general form for the parabolic expression is assumed (with undetermined constants—that can be determined by matching boundary conditions).



$$y = 2H_i\left(1-\frac{\theta_i}{\beta_i}\right)^2 = 2H_i\left(1-\frac{t_i}{T_i}\right)^2$$

- β_i is cam rotation needed to achieve displacement H_i *(y & θ are variables).*

Cam Dynamics …

c) Uniform acceleration and retardation/parabolic

- Displacement, velocity, and acceleration functions: A general form for the parabolic expression is assumed (with undetermined constants—that car be determined by matching boundary conditions).

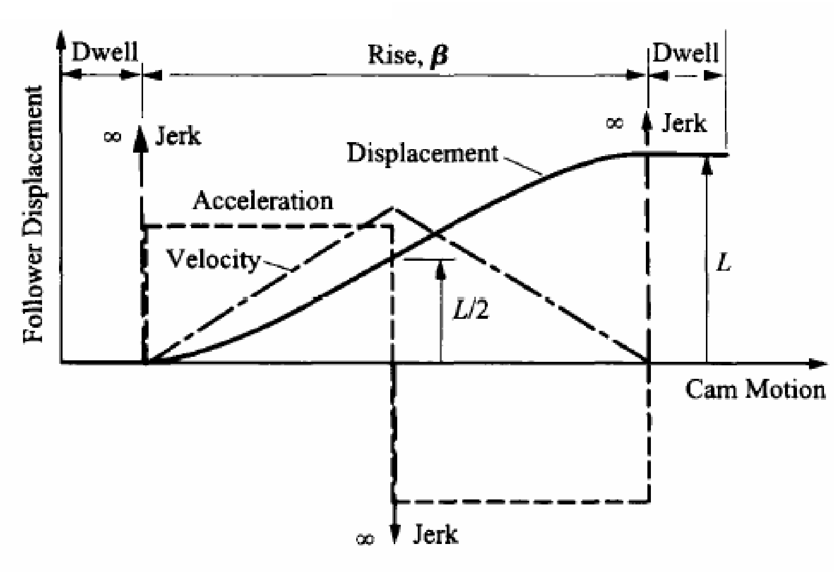
	Rise	Fall
For $0 < t < 0.5\,T\,(0 < \phi < 0.5\,\beta)$:		
Displacement:	$\Delta R_i = H_0 + 2H_i\left(\frac{t_i}{T_i}\right)^2 = H_0 + 2H_i\left(\frac{\phi_i}{\beta_i}\right)^2$	$\Delta R_j = H_F + H_j - 2H_j\left(\frac{t_j}{T_j}\right)^2 = H_F + H_j - 2H_j\left(\frac{\phi_j}{\beta_j}\right)^2$
Velocity:	$v_i = \frac{4H_i t_i}{T_i^2} = \frac{4H_i\omega\phi_i}{\beta_i^2}$	$v_j = \frac{-4H_j t_j}{T_j^2} = \frac{-4H_j\omega\phi_j}{\beta_j^2}$
Acceleration:	$a_i = \frac{4H_i}{T_i^2} = \frac{4H_i\omega^2}{\beta_i^2}$	$a_j = \frac{-4H_j}{T_j^2} = \frac{-4H_j\omega^2}{\beta_j^2}$
For $0.5\,T < t < T\,(0.5\,\beta < \phi < \beta)$:		
Displacement:	$\Delta R_i = H_0 + H_i - 2H_i\left(1 - \frac{t_i}{T_i}\right)^2 = H_0 + H_i + 2H_i\left(1 - \frac{\phi_i}{\beta_i}\right)^2$	$\Delta R_j = H_F + 2H_j\left(1 - \frac{t_j}{T_j}\right)^2 = H_F + 2H_j\left(1 - \frac{\phi_j}{\beta_j}\right)^2$
Velocity:	$v_i = \frac{4H_i}{T_i}\left(1 - \frac{t_i}{T_i}\right) = \frac{4H_i\omega}{\beta_i}\left(1 - \frac{\phi_i}{\beta_i}\right)$	$v_j = \frac{-4H_j}{T_j}\left(1 - \frac{t_j}{T_j}\right) = \frac{-4H_j\omega}{\beta_j}\left(1 - \frac{\phi_j}{\beta_j}\right)$
Acceleration:	$a_i = \frac{-4H_i}{T_i^2} = \frac{-4H_i\omega^2}{\beta_i^2}$	$a_j = \frac{4H_j}{T_j^2} = \frac{4H_j\omega^2}{\beta_j^2}$

- β_i is cam rotation needed to achieve displacement H_i *(y & θ are variables).*

Cam Dynamics …

c) Uniform acceleration and retardation/parabolic

- The shapes of each half of the displacement diagram are mirror-image parabolas. It has constant positive and negative accelerations.
- It has an abrupt change of acceleration at the end of the motion and at the transition point between the acceleration and deceleration halves.
- These result in abrupt changes in inertial forces, which typically cause undesirable vibrations.



- Therefore, this motion in its pure form is uncommon except for low-speed applications.

Cam Dynamics …

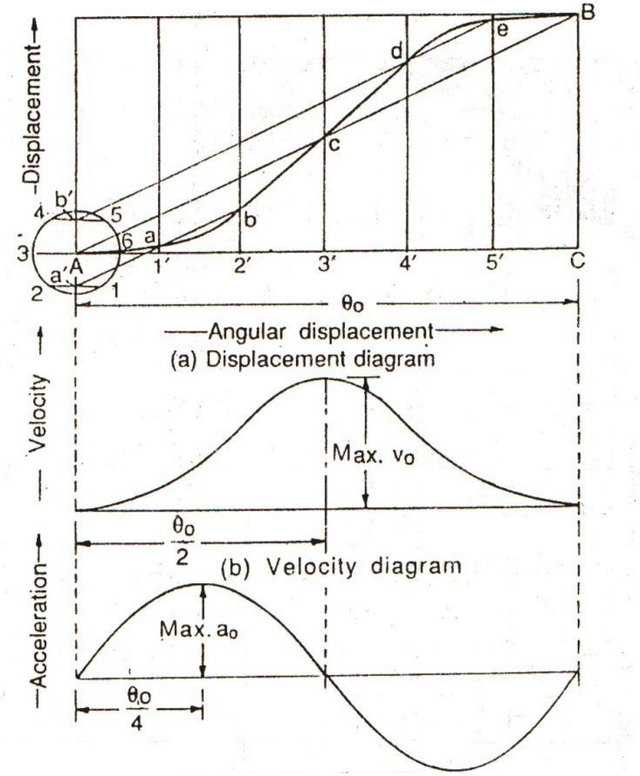
d) Cycloidal motion

- A cycloidal curve is the path traced by a point on a circle as the circle rolls on a straight line.

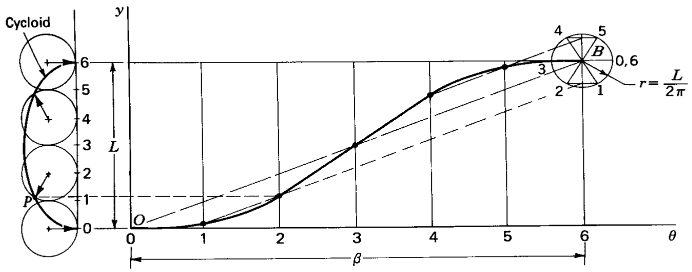
Cam Dynamics …

d) Cycloidal motion

- A cam can be designed for a cycloidal motion.



$$y = H_i\left(\frac{\theta}{\beta_i} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta_i}\right) = H_i\left(\frac{t}{T_i} - \frac{1}{2\pi}\sin\frac{2\pi t}{T_i}\right)$$



  - β is cam rotation needed to achieve displacement L *(y & θ are variables).*

Cam Dynamics …

d) Cycloidal motion

- Cam Follower Kinematics for Cycloidal Motion: Displacement, velocity and acceleration functions
- No acceleration discontinuities, therefore it can be applied to high speeds.

	Rise	Fall
Displacement:	$\Delta R_i = H_0 + H_i\left[\frac{t_i}{T_i} - \frac{1}{2\pi}\sin\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= H_0 + H_i\left[\frac{\phi_i}{\beta_i} - \frac{1}{2\pi}\sin\left(\frac{2\pi \phi_i}{\beta_i}\right)\right]$	$\Delta R_j = H_F + H_j\left[1 - \frac{t_j}{T_j} + \frac{1}{2\pi}\sin\left(\frac{2\pi t_j}{T_j}\right)\right]$ $= H_F + H_j\left[\frac{\phi_j}{\beta_j} - \frac{1}{2\pi}\sin\left(\frac{2\pi \phi_j}{\beta_j}\right)\right]$
Velocity:	$v_i = \frac{H_i}{T_i}\left[1 - \cos\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= \frac{H_i\omega}{\beta_i}\left[1 - \cos\left(\frac{2\pi \phi_i}{\beta_i}\right)\right]$	$v_j = \frac{-H_j}{T_i}\left[1 - \cos\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= \frac{-H_j\omega}{\beta_j}\left[1 - \cos\left(\frac{2\pi \phi_j}{\beta_j}\right)\right]$
Acceleration:	$a_i = \frac{2\pi H_i}{T_i^2}\left[\sin\left(\frac{2\pi t_i}{T_i}\right)\right]$ $= \frac{2\pi H_i\omega^2}{\beta_i^2}\left[\sin\left(\frac{2\pi \phi_i}{\beta_i}\right)\right]$	$a_j = \frac{-2\pi H_j}{T_j^2}\left[\sin\left(\frac{2\pi t_j}{T_j}\right)\right]$ $= \frac{-2\pi H_j\omega^2}{\beta_j^2}\left[\sin\left(\frac{2\pi \phi_j}{\beta_j}\right)\right]$

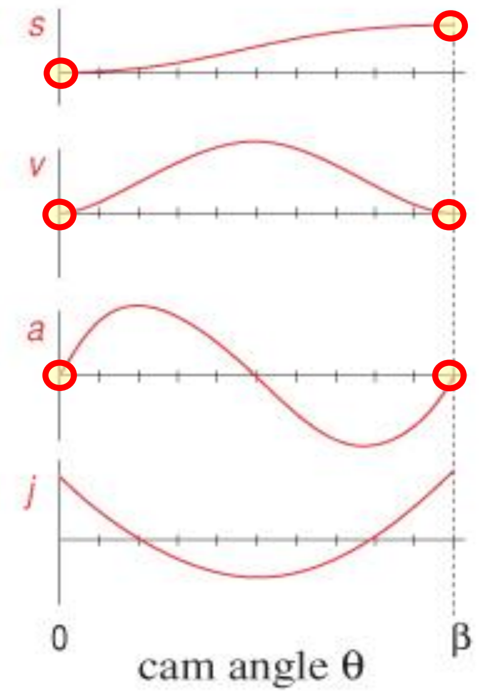
Cam Dynamics …

e) General polynomial motion

- We can also choose polynomials for cam functions
- General form:

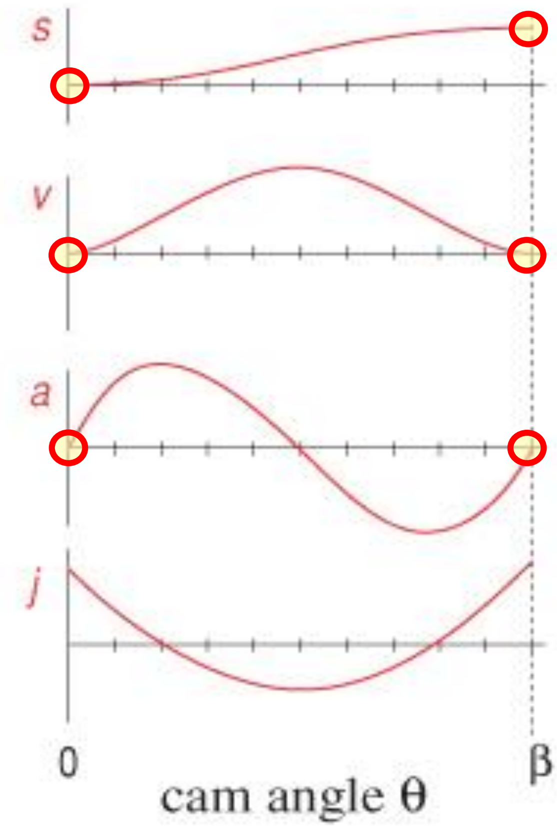
$$\boxed{s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots + C_n x^n}$$

- where $x=\theta/\beta$ or t/T
- Choose the number of boundary conditions (BC's) to satisfy the fundamental law of cam design



Cam Dynamics …

e) 3-4-5 Polynomial



- Substituting
  - where $x=\theta/\beta$ or t/T
- Applying boundary conditions
  - $@\theta=0,\ s=0, v=0, a=0$
  - $@\theta=\beta,\ s=h, v=0, a=0$
- Six boundary conditions, so order 5 since C_0 term
- We get

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5$$

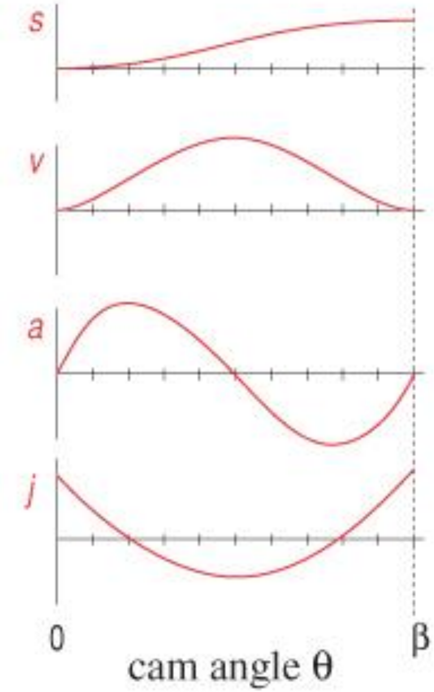
Cam Dynamics …

e) 3-4-5 Polynomial

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5$$

$$v = \frac{1}{\beta}\left[C_1 + 2C_2\left(\frac{\theta}{\beta}\right) + 3C_3\left(\frac{\theta}{\beta}\right)^2 + 4C_4\left(\frac{\theta}{\beta}\right)^3 + 5C_5\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a = \frac{1}{\beta^2}\left[2C_2 + 6C_3\left(\frac{\theta}{\beta}\right) + 12C_4\left(\frac{\theta}{\beta}\right)^2 + 20C_5\left(\frac{\theta}{\beta}\right)^3\right]$$



@θ=0, $s=0=C_0$ $v=0=C_1/\beta$ $a=0=2C_2/\beta^2$

$C_0=0$ $C_1=0$ $C_2=0$

@$\theta=\beta$, $s=h= C_3+C_4+C_5$, $v=0=2C_3+3C_4+5C_5$; $a=0= 6C_3+12C_4+20C_5$

Solve the 3 equations to get

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

Cam Dynamics …

e) 3-4-5 and 4-5-6-7 polynomial

- 3-4-5 polynomial

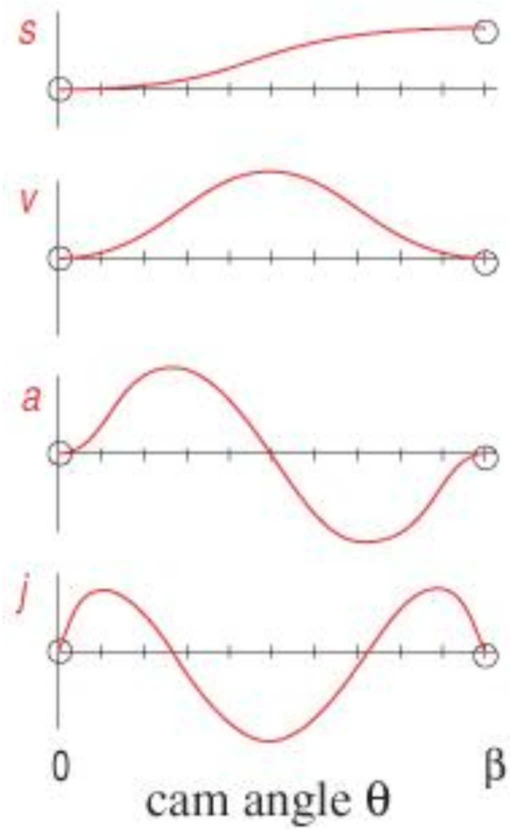
  Similar in shape to cycloidal; discontinuous jerk

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

- 4-5-6-7 polynomial: set the jerk to be zero at 0 and *b.* Has continuous jerk, but everything else is larger

$$s = h\left[35\left(\frac{\theta}{\beta}\right)^4 - 84\left(\frac{\theta}{\beta}\right)^5 + 70\left(\frac{\theta}{\beta}\right)^6 - 20\left(\frac{\theta}{\beta}\right)^7\right]$$

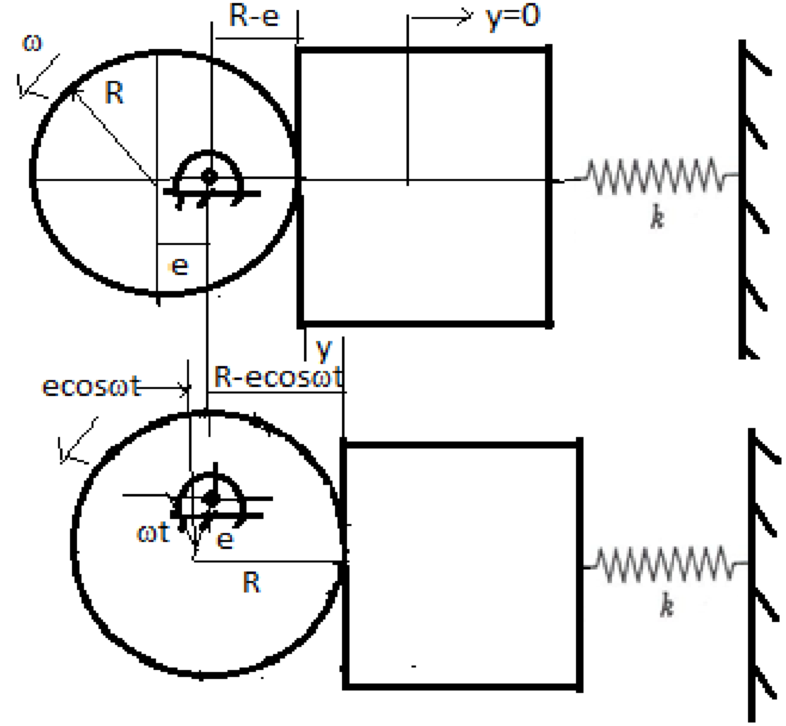
4-5-6-7 Polynomial



Cam Dynamics …

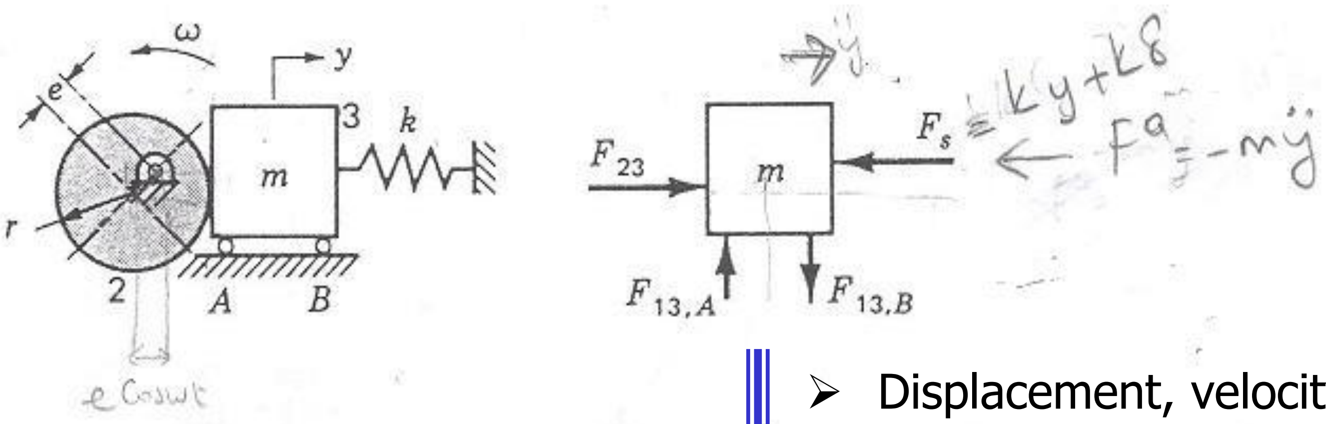
Analysis of an Eccentric Cam

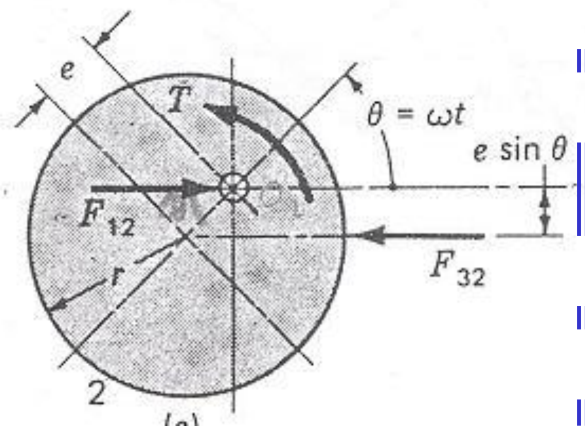
- An eccentric cam is a circular disc with a camshaft hole drilled off-center
- The distance *e* between the center of the disc and the center of the cam is known as eccentricity.
- The components of a plate cam mechanism are:
  - A plate cam
  - A flat face follower mass, and
  - A retaining spring



Cam Dynamics …

Analysis of an Eccentric Cam





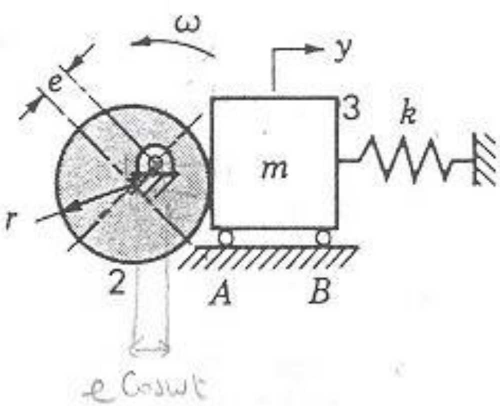
- Displacement, velocity and acceleration

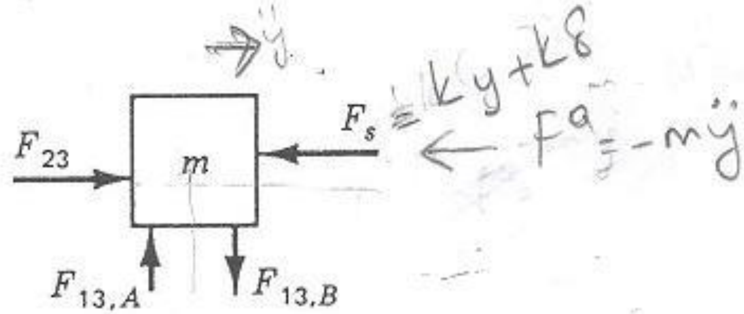
$$y = e - e.\cos(\omega t) = e\left[1-\cos(\omega t)\right]$$
$$\dot{y} = e\omega.\sin(\omega t)$$
$$\ddot{y} = e\omega^2.\cos(\omega t)$$

Where, k = spring constant; e = eccentricity; P = preload, δ = elongation; F_{23} = Force exerted by the follower; T = Torque ; ω = angular speed of cam

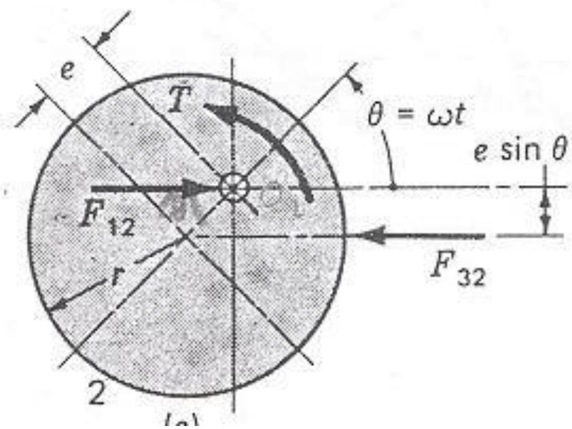
Cam Dynamics …

Analysis of an Eccentric Cam (Determination of the force exerted by the follower)





- FBD of the follower



$$\sum F_y = F_{23} - F_s - m\ddot{y} = 0$$

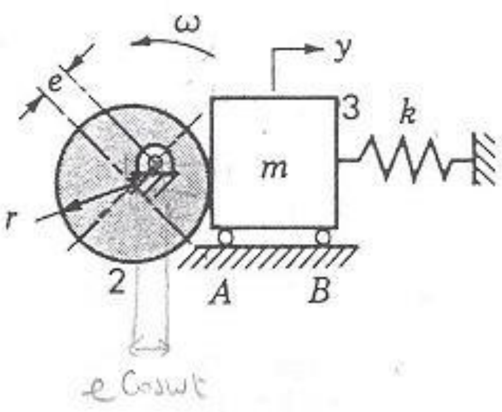
$$\sum F_y = F_{23} - k(y+\delta) - m\ddot{y} = 0$$

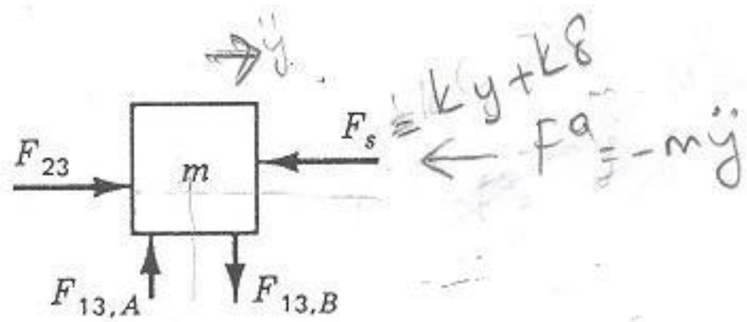
$$F_{23} = (ke + P) + (m\omega^2 - k).e.\cos(\omega t)$$

Where, k = spring constant; e = eccentricity; P = preload, δ = elongation; F_{23} = Force exerted by the follower; T = Torque ; ω = angular speed of cam

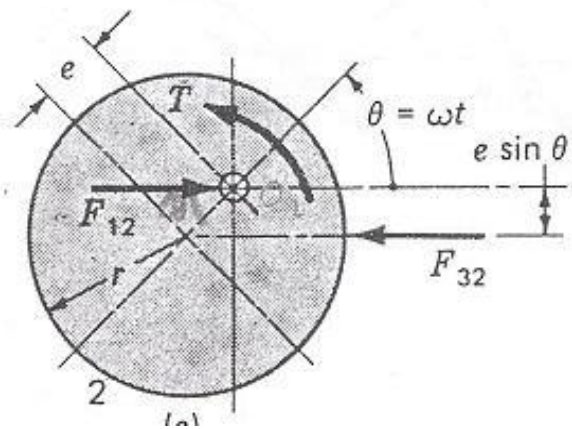
Cam Dynamics …

Analysis of an Eccentric Cam (Determination of Torque)





- FBD of the cam



$$T = F_{23}.e.\sin(\omega t)$$

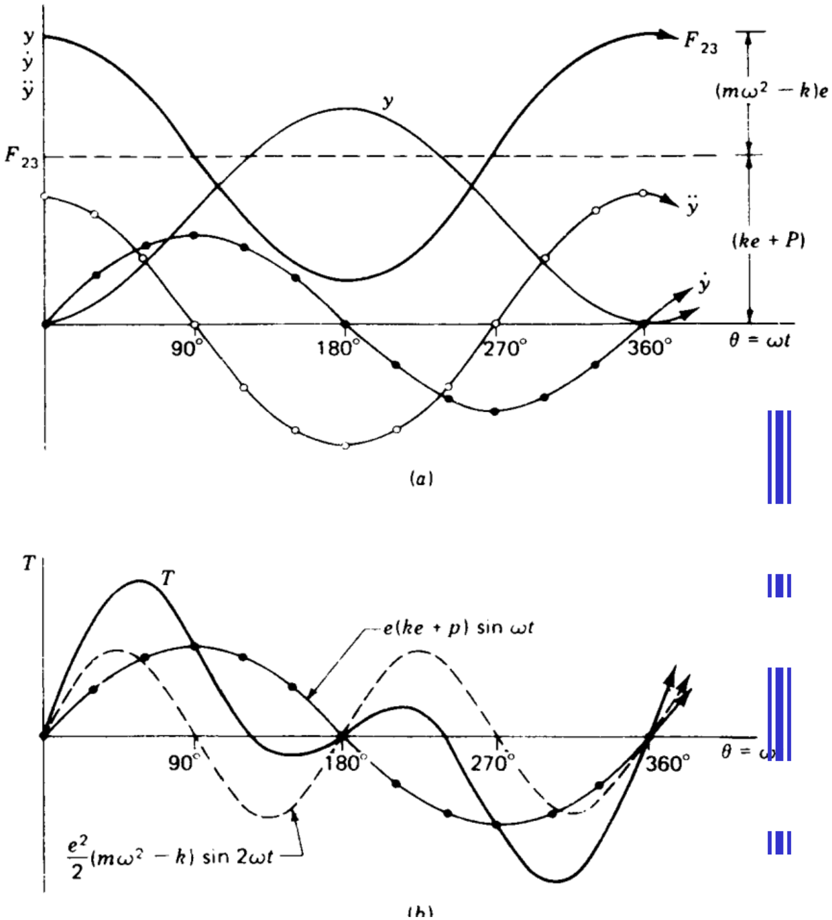
$$T = \left[(ke+P)+(m\omega^2-k).e.\cos(\omega t).e.\sin(\omega t)\right]$$

$$T = e(ke+P)\sin(\omega t)+\frac{e^2}{2}(m\omega^2-k).\sin(2\omega t)$$

Where, k = spring constant; e = eccentricity; P = preload, δ = elongation; F_{23} = Force exerted by the follower; T = Torque ; ω = angular speed of cam

Cam Dynamics …

Analysis of an Eccentric Cam (Plots)



a) Plot of Displacement, Velocity, Acceleration and Contact Force for an Eccentric Cam System

b) Graph of Torque Components and Total Cam-Shaft Torque

In the torque characteristic, the (+) area is equal to (-) area. i.e. Energy required to drive the follower in the forward direction is recovered when the follower returns. A flywheel may be used on the cam shaft to handle this fluctuating energy requirement.

Cam Dynamics …

About Jump

- To prevent jump, increase the term ke+P, by increasing preload P, or spring constant k, or both.

- Here, jump begins at ωt=180 with cos ωt=-1; F_{23}=0

$$F_{23} = (ke + P) + (m\omega^2 - k).e.\cos(\omega t) = 0$$

$$\omega = \sqrt{\frac{2ke + P}{m.e}}$$

This is the critical speed

- Jump will not occur if,

$$P > e.(m\omega^2 - 2k)$$

Where, k = spring constant; e = eccentricity; P = preload, δ = elongation; F_{23} = Force exerted by the follower; T = Torque ; ω = angular speed of cam

Cam Dynamics …

Unbalance

- Disk cam produces unbalance because its mass is not symmetrical with the axis of rotation.
- There are 2 sets of vibratory forces, namely –
  - one due to eccentric cam mass, and;
  - the other reaction of the follower against the cam

Spring Surge

- The vibration of the retaining spring, is called ‘spring surge’
- Basically, springs may themselves vibrate when subjected to rapidly varying forces; e.g., poorly designed automotive valve springs operating near the critical frequency range permits the valve to open for short intervals during the period the valve it is supposed to be closed.

Cam Dynamics …

Windup

- Wind up is the tendency of shaft to twist (i.e., to windup) due to varying torque.
- In a cam-shaft, the shaft exerts torque on the cam during a portion of a cycle and the cam exerts torque of the shaft during another portion.
- The varying torque may cause the shaft to twist or wind up, as the torque increases during follower rise.

Lesson 7 Revision Problems

1) Define or explain the following cam dynamics terms or phenomena: (a) displacement, (b) velocity, (c) acceleration, (d) jerk, (e) windup (f) unbalance, (g) spring surge.

2) Show how velocity, acceleration and jerk is determined for the following follower motion schemes:

   a) Uniform motion (constant velocity)

   b) Simple harmonic motion

   c) Uniform acceleration and retardation/parabolic motion

   d) Cycloidal motion

   e) General polynomial motion

End…

Any Questions?