Lesson 5

Dynamic Analysis of Reciprocating Engines

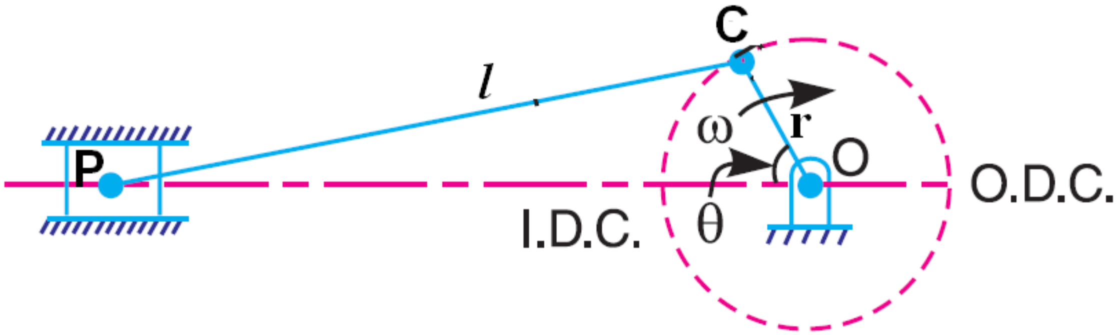
Dynamic Analysis of Reciprocating Engines ...

Force Analysis in IC Engine Mechanism

- The study of forces in IC engine mechanism are classified into two types as follows :
  - **Static force analysis**
  - **Dynamic force analysis**
- In **static force analysis**, we do not consider the effect of inertia forces arising <u>due to the mass of the connecting rod.</u>
- In **dynamic force analysis**, <u>we also consider</u> the effect of inertia forces caused due to the mass of connecting rod.
- The force analysis can be done both by analytical and graphical methods.

Dynamic Analysis of Reciprocating Engines …

Static Force Analysis of IC Engine Mechanism

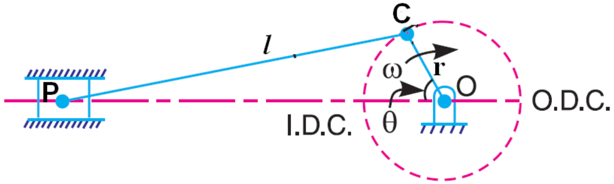


Slider crank mechanism.

- Consider an IC engine mechanism shown in Fig. in which the crank OC rotates at an angular speed of ω in clockwise direction.
- … at an instant it is inclined at angle θ from I.D.C.

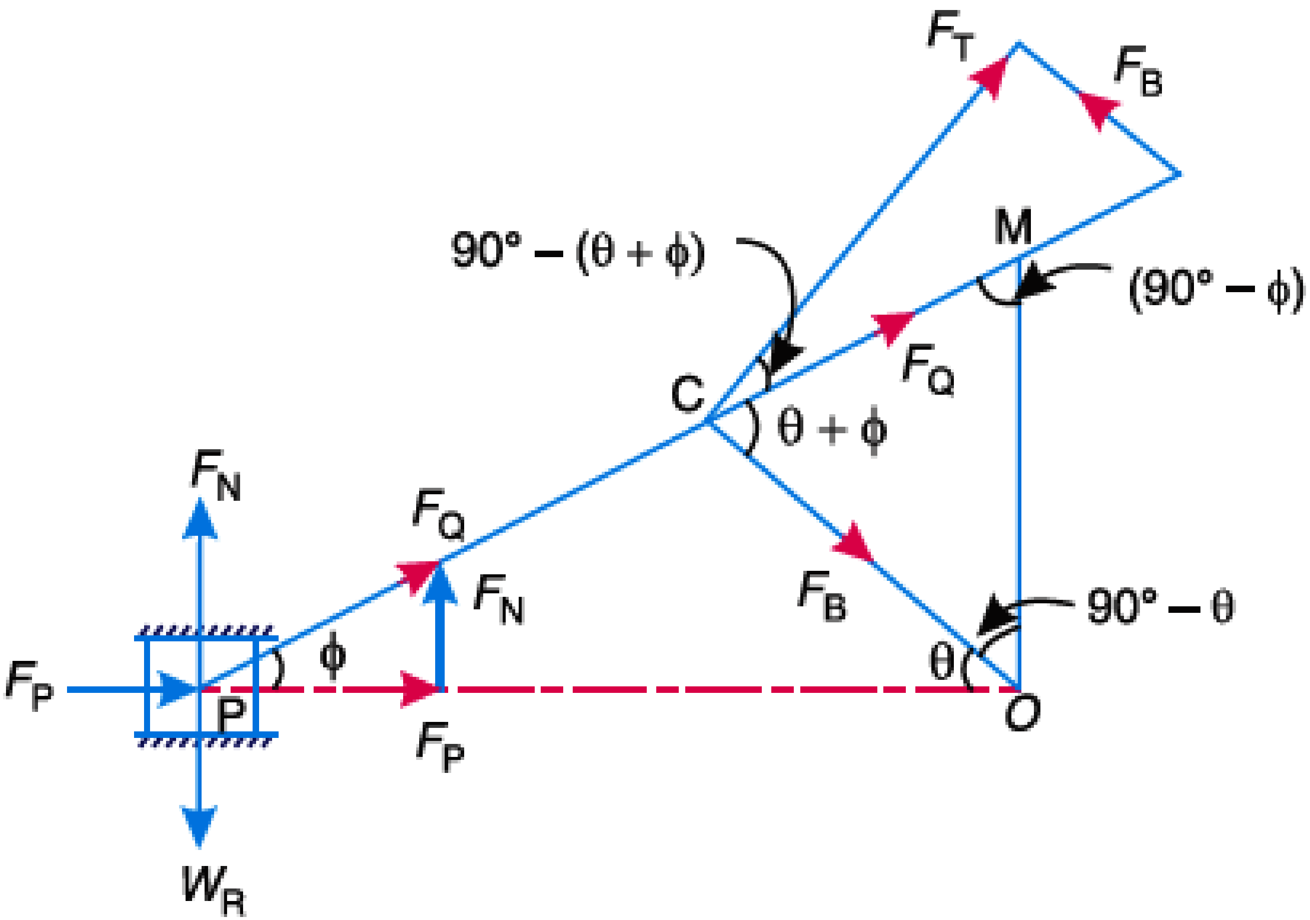
Dynamic Analysis of Reciprocating Engines …

- Let,
  - **r** = Radius of crank,
  - **l** = Length of connecting rod,
  - **n** = Obliquity ratio = l/r
  - $\mathbf{m_R}$ = Mass of the reciprocating parts, e.g. piston, crosshead pin or gudgeon pin etc., in kg,
  - $\mathbf{W_R}$ = Weight (in N) of the reciprocating parts = $m_R.g$
  - **m** = Mass of the connecting rod,
  - **θ** = Angle made by crank with I.D.C.,
  - **Φ** = Angle made by connecting rod with line of reciprocation (i.e. line OP) when line is inclined at angle θ



Slider crank mechanism.

Dynamic Analysis of Reciprocating Engines …



Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

➢ Piston effort

- Is the net force acting on the piston or crosshead pin, along the line of stroke… denoted by F_P (in the Fig).

- The acceleration of the reciprocating parts,

$$a_R = a_P = \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

- Accelerating force or inertia force of the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

- Therefore, Piston effort, F_P = Net load on the piston ± Inertia force

$= F_L \pm F_I$ *…(Neglecting frictional resistance)*

$= F_L \pm F_I - R_F$ *…(Considering frictional resistance)*

where, R_F = *Frictional resistance.* The –ve sign is used when the piston is accelerated, and +ve sign is used when the piston is retarded.

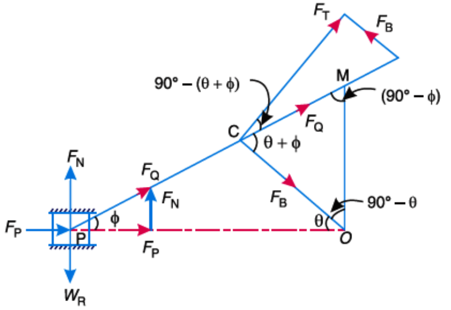
Dynamic Analysis of Reciprocating Engines …

- In a reciprocating steam engine, net load on the piston,

$$F_L = p_1 A_1 - p_2 A_2 = p_1 A_1 - p_2 (A_1 - a)$$

where,

  - p_1, A_1 = Pressure and cross-sectional area on the back end side of the piston,
  - p_2, A_2 = Pressure and cross-sectional area on the crank end side of the piston,
  - a = Cross-sectional area of the piston rod.



Forces on the reciprocating parts of an engine.

- **Notes :**
  - If 'p' is the net pressure of steam or gas on the piston and D is diameter of the piston, then in case of a vertical engine,

Net load on the piston, F_L = Pressure × Area $= p \times \frac{\pi}{4} \times D^2$

Piston effort, $F_P = F_L \mp F_I \pm W_R - R_F$

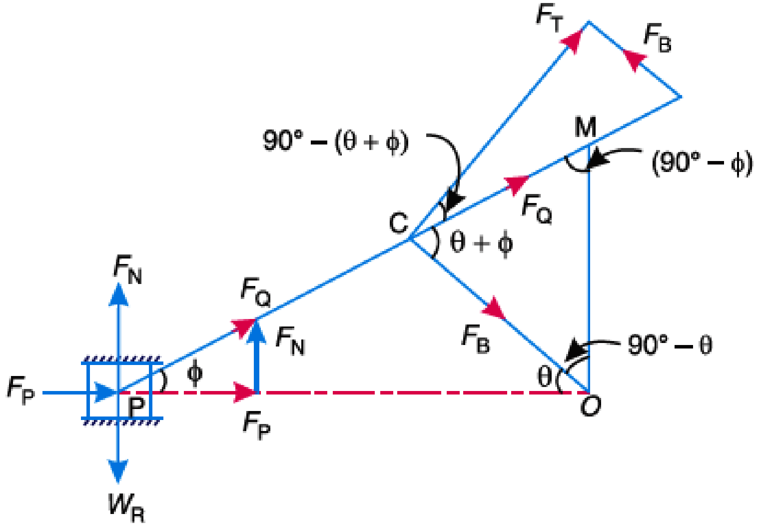
Dynamic Analysis of Reciprocating Engines …

- **Force acting along the connecting rod.**
  - It is denoted by F_Q in Fig. From the geometry of the figure, we find that

$$F_Q = \frac{F_P}{\cos\phi}$$

We know that $$\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$\therefore \qquad F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

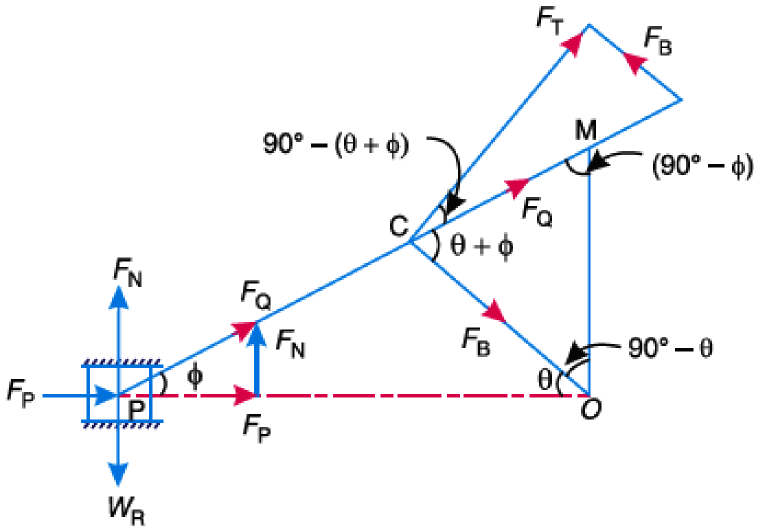


Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

- **Thrust on the sides of the cylinder walls or normal reaction on the guide bars.**

  It is denoted by F_N in Fig. From the figure, we find that



Forces on the reciprocating parts of an engine.

$$F_Q = \frac{F_P}{\cos\phi}$$

$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi} \times \sin\phi = F_P \tan\phi \qquad \ldots\left[\because F_Q = \frac{F_P}{\cos\phi}\right]$$

Dynamic Analysis of Reciprocating Engines …

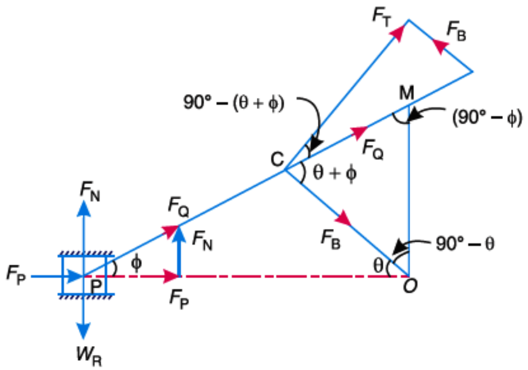
➢ Crank-pin effort and thrust on crank shaft bearings

- The force acting on the connecting rod F_Q may be resolved into two components, one perpendicular to the crank and the other along the crank.
- The component of F_Q perpendicular to the crank is known as crank-pin effort and it is denoted by F_T in Fig.
- The component of F_Q along the crank produces a thrust on the crank shaft bearings and it is denoted by F_B in Fig.
- Resolving F_O perpendicular to the crank,

$$F_T = F_Q \sin(\theta + \phi) = \frac{F_P}{\cos\phi} \times \sin(\theta + \phi)$$

- and resolving *FQ along the crank,*

$$F_B = F_Q \cos(\theta + \phi) = \frac{F_P}{\cos\phi} \times \cos(\theta + \phi)$$



Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

➢ Crank effort or turning moment or torque on the crank shaft

- The product of the crankpin effort (F_T) and the crank radius (r) is known as crank effort or turning moment or torque on the crank shaft.
- Mathematically, Crank effort,

$$T = F_T \times r = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r$$

$$= \frac{F_P (\sin\theta \cos\phi + \cos\theta \sin\phi)}{\cos\phi} \times r$$

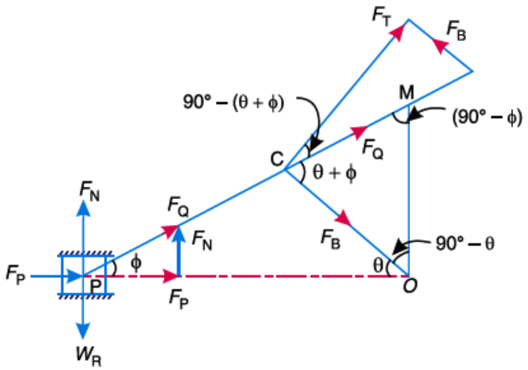
$$= F_P \left(\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi} \right) \times r$$

$$= F_P (\sin\theta + \cos\theta \tan\phi) \times r \qquad \ldots(i)$$

We know that $l \sin\phi = r \sin\theta$

$$\sin\phi = \frac{r}{l} \sin\theta = \frac{\sin\theta}{n} \qquad \ldots\left(\because n = \frac{l}{r}\right)$$

and $$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$



Forces on the reciprocating parts of an engine.

Dynamic Analysis of Reciprocating Engines …

$$\therefore \qquad \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{n} \times \frac{n}{\sqrt{n^2 - \sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Substituting the value of $\tan\phi$ in equation (*i*), we have crank effort,

$$T = F_P \left(\sin\theta + \frac{\cos\theta \sin\theta}{\sqrt{n^2 - \sin^2\theta}} \right) \times r$$

$$= F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right) \qquad \ldots(ii)$$

$\ldots(\because\ 2\cos\theta\sin\theta = \sin 2\theta)$

Note: Since $\sin^2\theta$ is very small as compared to n^2 therefore neglecting $\sin^2\theta$, we have,

Crank effort, $$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2n} \right) = F_P \times OM$$

We have seen in Art. 15.8, that

$$OM = r\left(\sin\theta + \frac{\sin 2\theta}{2n} \right)$$

Therefore, it is convenient to find *OM* instead of solving the large expression.

Lesson 5 Revision Problems

1) How is the study of forces and torque in IC engine mechanism classified?
2) What is piston effort? Explain how to determine piston effort.
3) How is net load on the piston in a reciprocating steam engine determined?
4) Explain how is thrust on the sides of the cylinder walls (or normal reaction on the guide bars) determined?
5) What is crank effort? Explain how to determine crank effort?
6) Use a suitable sketch to explain how to determine crank-pin effort and thrust on crank shaft bearings
7) Use a suitable sketch to show how to determine force acting along the connecting rod.

End…

Any Questions?