Lesson 2

# Degrees of Freedom

# Degrees of Freedom (DoF)…

## Definition

- DoF (also known as mobility) of a rigid body is defined as the number of independent movements that at the body has.
- To determine DoF of a rigid body, we must consider how many distinct ways it can be moved.
- DoF is needed to uniquely define position of a system in space at any instant of time.

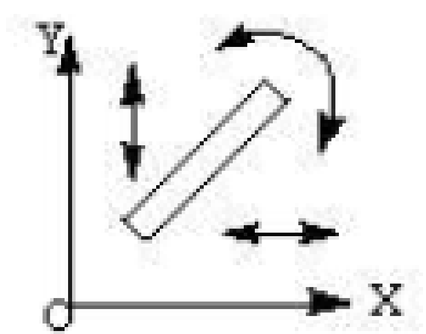
## Types of Motion

- Pure rotation: the body possesses one point (center of rotation) that has no motion with respect to the "stationary" frame of reference. All other points move in circular arcs.
- Pure translation: all points on the body describe parallel (curvilinear or rectilinear) paths.
- Complex motion: a simultaneous combination of rotation and translation... frame of reference (ground)

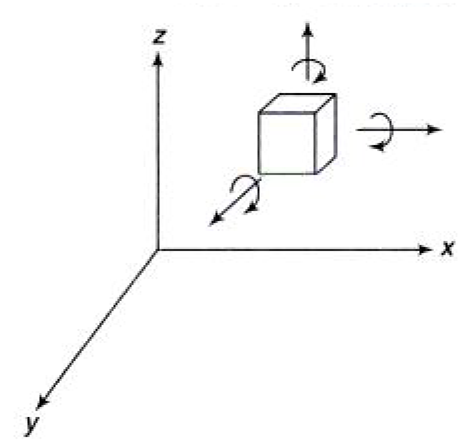
# Degrees of Freedom (DoF)…

## DoF of a rigid body in a Plane

- For a plane (a 2D plane), e.g., a computer screen, there are 3 DoF, i.e.. The body can be translated along x @ y – axes and rotated about z-axis.




## DoF of a rigid body in Space




- A constrained rigid body moving in space can be:
  - Translated along along x, y & z
  - Rotated about x, y, & z
- Therefore, a rigid body in space possesses 6 DoF

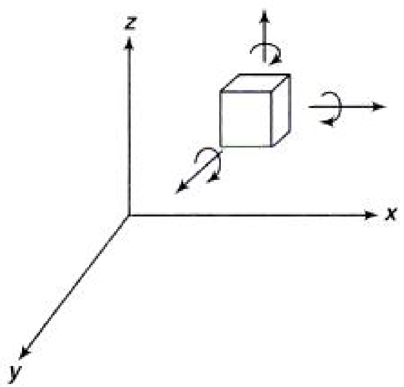
# Degrees of Freedom (DoF)…

## DoF of a Pair (e.g., connected 2 rigid bodies)

- The connection of a link ( a rigid body) with another imposes certain constrains on their relative motion:
  - Note that the number of restraints can never be 0 (i.e., in this case no joint!) or 6 (i.e., in this case, joint becomes a solid!).
  - Therefore, DoF or mobility of a pair (m) is defined as the number of independent relative motions (both rotational or rotational) that a pair can have , i.e.,

$$m = 6 - r$$

where r is the number of restraints.

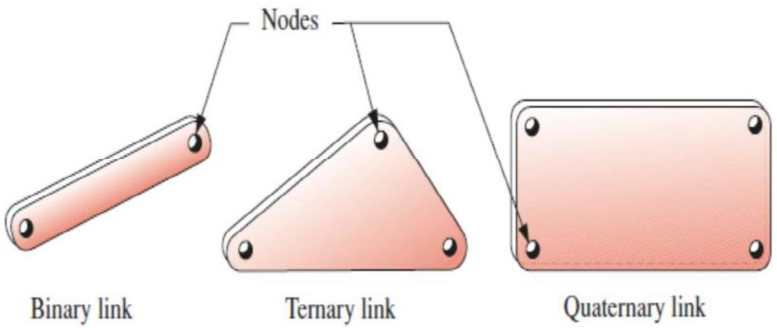


# Degrees of Freedom (DoF)…

## DoF of a Mechanism

- A mechanism can also have several DoFs.
- The DoF of a mechanism is decided by the DoF of the links constituting that mechanism.

Linkages (are made up of links and joints) are the basic building blocks of all common forms of mechanisms (e.g., cams, gears, belts, chains). Links are rigid member having nodes (attachment points)




Recall that Joint: connection between two or more links (at their nodes) which allows motion; (Joints also called kinematic pairs)

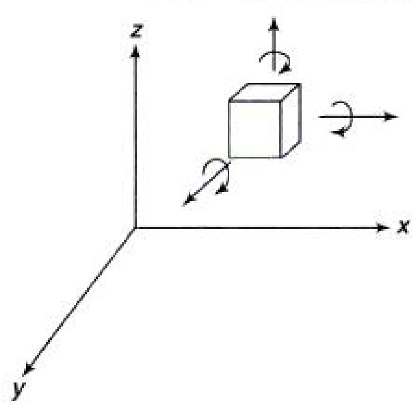
# Degrees of Freedom (DoF)…

## DoF of a Mechanism

We can classify mechanisms in two general categories, as follows :

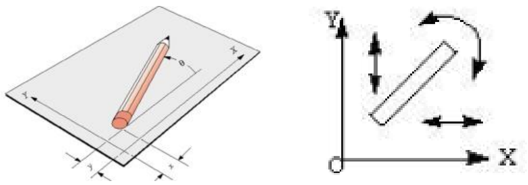
1) Spatial mechanism:

- The complete <u>motions cannot be represented in a single plane</u>, i.e., to describe the motion of such mechanisms, more than one plane would be required. They have three dimensional motion paths.
- Examples: Robot arm, Cranes, etc.




2) Planer mechanism:

- The complete <u>motion paths of the mechanism can be represented on a single plane</u>., i.e., the entire mechanism can be represented on a sheet of paper.




Note: we also have spherical mechanisms (composed of mechanical links, hinges, and sliding joints) designed to produce complex 3D motions.

# Mobility and DoF…

- Mobility and DoF are essentially the same with very little difference.

- DoF is the number of independent co-ordinates required to define the position of each link, in a mechanism, while mobility is the number of independent input parameters that are to be controlled so that the mechanism can take up a particular position.
- Kutzbach's (also referred as Grübler's Criterion in some literature) is widely used to determine DoF of mechanisms.

# Mobility and DoF…

## Kutzbach Criterion (Generic)

- Degrees of freedom (DoF) of a mechanism in space can be determined as follows :

  Let, L = Total number of links in a mechanism

  m = DoF of a mechanism/mobility

- In a mechanism one link should be fixed. Therefore total number of movable links in a mechanism is (L– 1).

- Thus, total number of DoF of (L – 1) movable links is,

$$m = 6 (L - 1)$$

# Mobility and DoF...

## Kutzbach Criterion (Generic)

Let, $j_1$ = Number of joints/pairs having 1 DoF ;
$j_2$ = Number of joints/pairs having 2 DoF ;
$j_3$ = Number of joints/pairs having 3 DoF ;
$j_4$ = Number of joints/pairs having 4 DoF ;
$j_5$ = Number of joints/pairs having 5 DoF ;
$j_6$ = Number of joints/pairs having 6 DoF ;

We know that,

- Any pair having 1 DoF will impose 5 restrains on the mechanism, which reduces its total degree of freedom by 5 $j_1$.
- Any pair having 2 DoF will impose 4 restrains on the mechanism, which reduces its total degree of freedom by 4 $j_2$.
- Similarly for pair having 3 DoF, 4 DoF and 5 DoF will reduces its total degree of freedom by $3j_3$, 2 $j_4$ and 1 $j_5$ respectively and for pair having 6 DOF will impose zero restrains on mechanism, which reduces its total degree of freedom by zero.

# Mobility and DoF…

## Kutzbach Criterion (Generic)

- Therefore, in a mechanism if we consider the links having 1 to 6 DoF, the total number of degree of freedom of the mechanism considering all restrains will becomes,

$$m = 6\,(L - 1) - 5\,j_1 - 4\,j_2 - 3\,j_3 - 2\,j_4 - 1\,j_5 - 0\,j_6$$

- The above equation is the general form of Kutzbach criterion. This is applicable to any type of mechanism including a spatial mechanism.

# Mobility and DoF…

## Kutzbach Criterion (Planar Mechanism)

- For a planar mechanism, each link has 3 DoF before any of the joints are connected. Not connecting the fixed link, a *L* link planar mechanism has

$$m = 3\,(L - 1)$$

- If a joint which has one DoF – ($j_1$) (e.g., a revolute pair) is connected, it provides 2 constraints between the connected links.
- If a 2 DoF pair ($j_2$) is connected, it provide one constraint.
- When the constraints for all joints/pairs are subtracted, we find the mobility/DoF of the connected mechanism

$$m = 3\,(L - 1) - 2j_1 - j_2$$

- The above equation is the Kutzbach criterion applicable to any planar mechanism.

# Mobility and DoF…

## Kutzbach Criterion (Planar Mechanism)

- *If $m > 0$*; the system is a mechanism, with *m* degrees of freedom, and the mechanism will exhibit relative motion.
- *If $m = 1$*; the mechanism can be driven by a single input motion.
- *If $m = 2$*; then two separate input motions are necessary to produce constrained motion for the mechanism.
- *If $m = 0$*; motion is impossible. The system has enough constraints at the joints necessary to ensure equilibrium.
- *If $m = -1$ or less*; then there are redundant constraints in the chain and it forms a statically indeterminate structure. No motion is possible…basically the links have more constraints than are needed to maintain equilibrium (TRUSS).

# Mobility and DoF…

## Simplified Kutzbach/Grübler's Criterion

- Some literature refer to this equation as Kutzbach's Criterion and it simplified version (where j2 = 0) as Grüblers Criterion, i.e.,

$$m = 3(L - 1) - 2j_1 - j_2$$
$$m = 3(L - 1) - 2j_1 - 0$$
$$m = 3(L - 1) - 2j_1$$

(Many authors make no distinction between Kutzback and Grüblers criterion)

# Determining DoF…

Steps involved in determining mobility of mechanisms….

- Count number of elements/links: *L*
- Count number of single DoF pairs: $j_1$
- Count number of two DoF pairs: $j_2$
- Apply the equation below …

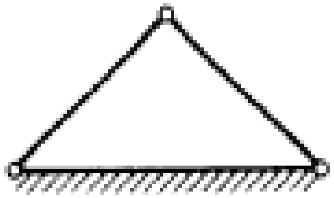
$$m = 3(L - 1) - 2j_1 - j_2$$

- Classify the system into mechanism, structure, or statically indeterminate system

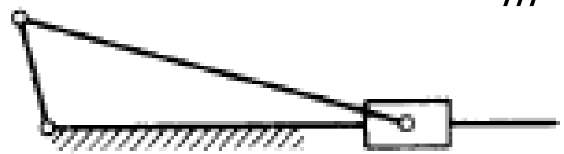
# Determining DoF…

Apply Kutzback/Grübler criterion … the equation below …

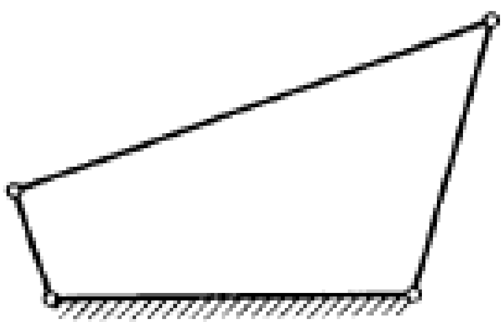
$$m = 3(L - 1) - 2j_1 - j_2$$



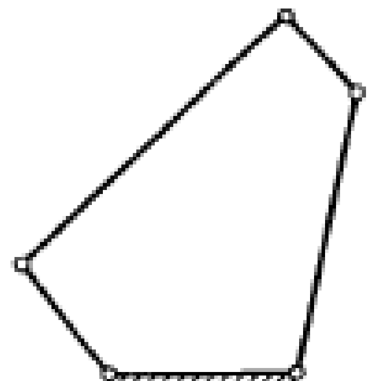
$n = 3, j_1 = 3$
$j_2 = 0, m = 0$
(a)



$n = 4, j_1 = 4,$
$j_2 = 0, m = 1$
(b)



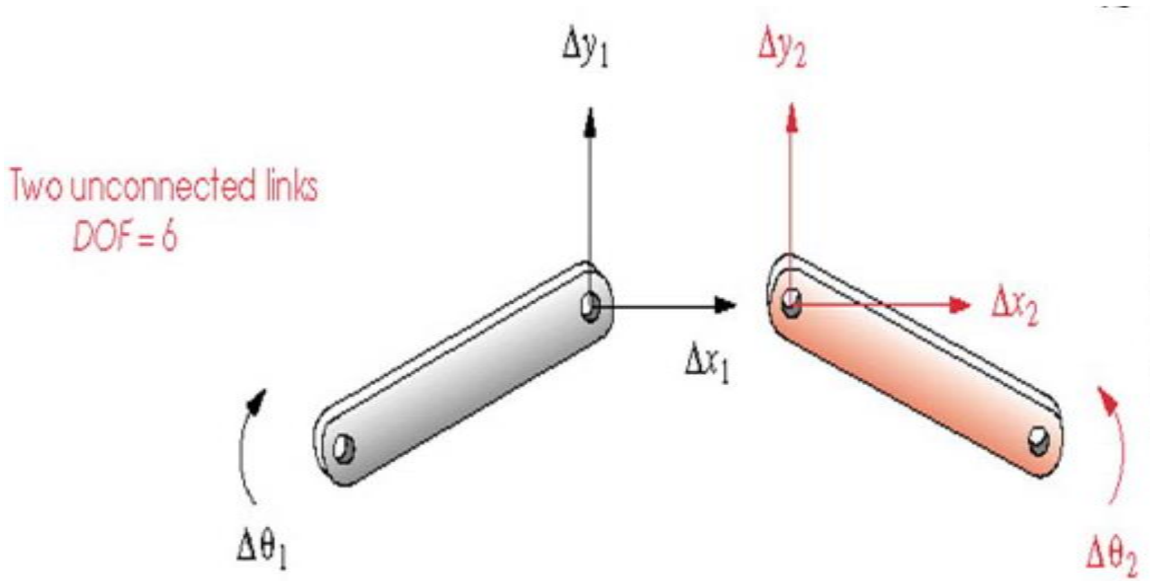
$n = 4, j_1 = 4,$
$j_2 = 0, m = 1$
(c)



$n = 5, j_1 = 5,$
$j_2 = 0, m = 2$
(d)

- *If $m > 0$*; the system is a mechanism, with *m* degrees of freedom, and the mechanism will exhibit relative motion.
- *If $m = 1$*; the mechanism can be driven by a single input motion.
- *If $m = 2$*; then two separate input motions are necessary to produce constrained motion for the mechanism.
- *If $m = 0$*; motion is impossible. The system has enough constraints at the joints necessary to ensure equilibrium.
- *If $m = -1$ or less*; then there are redundant constraints in the chain and it forms a statically indeterminate structure. No motion is possible…basically the links have more constraints than are needed to maintain equilibrium (TRUSS).

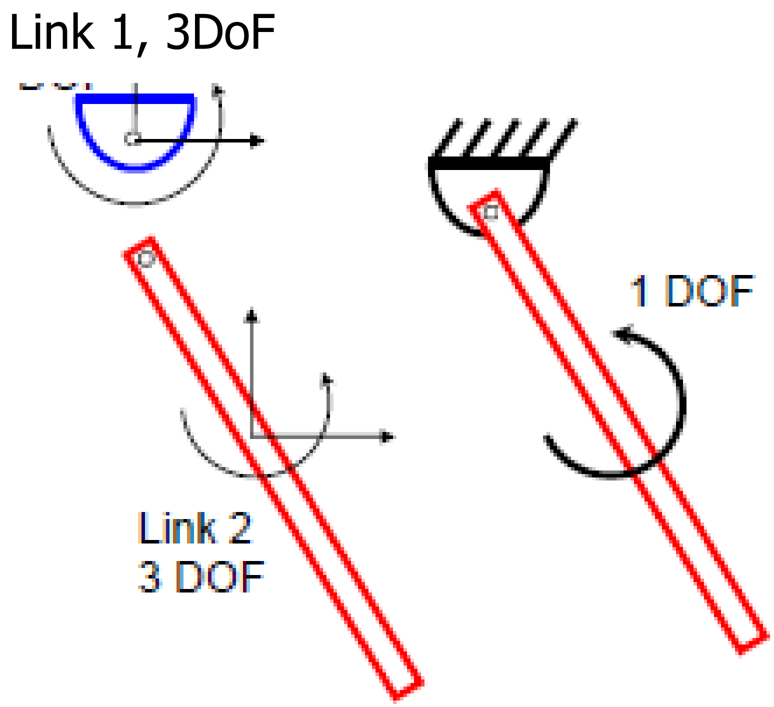
# Determining DoF…




- Therefore, for two unconnected links: 6 DoF (each link has 3 DoF)

# Determining DoF…

Consider the mechanism below ….




- Kutzbach's/Gruebler's equation for planar mechanisms:

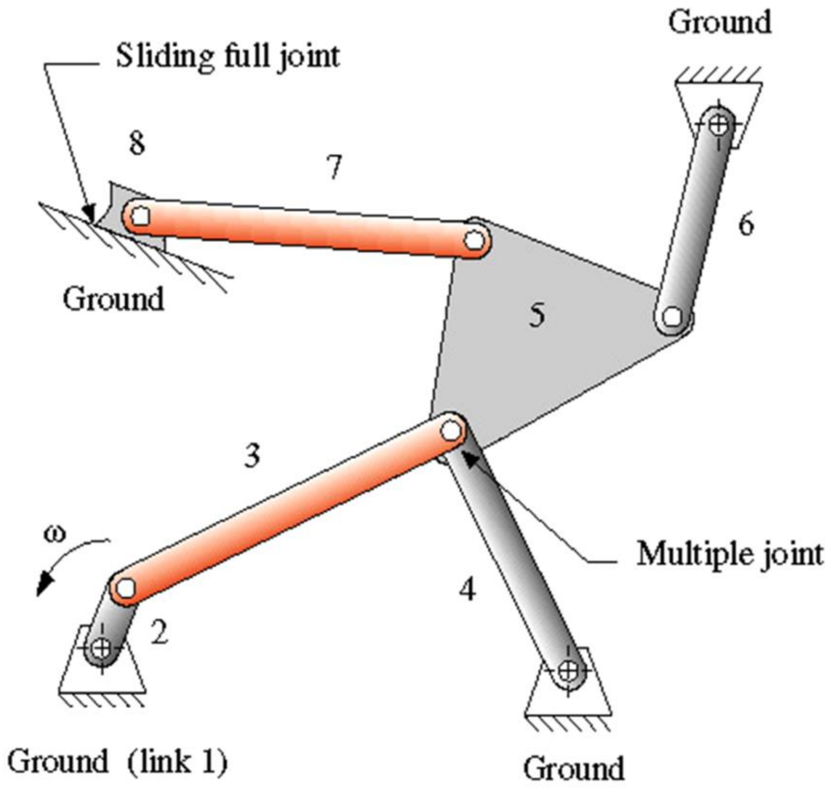
$$m = 3(L-1)-2j_1$$

where, L: number of links, (=2); and $j_1$: number of full joints (=1).

- Therefore, this mechanism has: 1 DoF

# Determining DoF…

Consider the mechanism below ….




- Kutzbach's/Gruebler's equation for planar mechanisms:

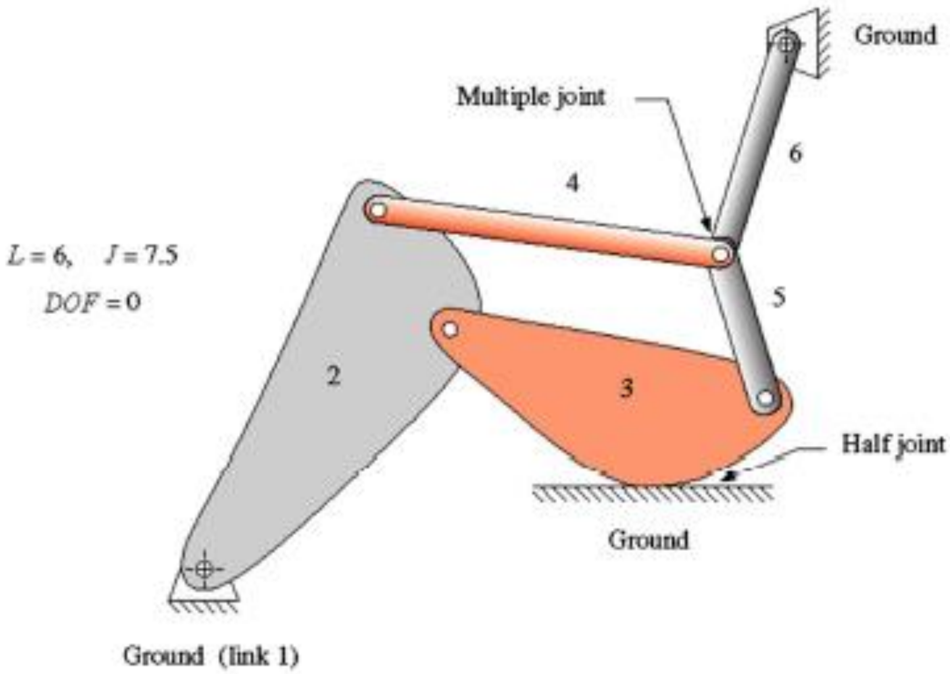
$$m= 3(L-1)-2j_1$$

where, L: number of links, (=8); and $j_1$: number of full joints (=10).

- Therefore, a cylindrical joint has: 1 DoF (thus it is a mechanism, its elements can move to perform the intended function)

# Determining DoF…

Consider the mechanism below ….




- Kutzbach's/Gruebler's equation for planar mechanisms:

$$m = 3(L\text{-}1)\text{-}2j_1\text{-}j_2$$
$$m = 3(6\text{-}1)\text{-}2x7\text{-}1$$
$$m = 0$$

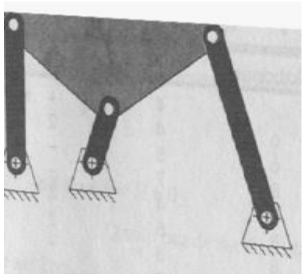
where, L: number of links, (=6); $j_1$, number 1 DoF joints (=7), $j_2$, number of 2 or more DoF joints (=1).

- Therefore, this mechanism has: 0 DoF (motion is impossible. The system has enough constraints at the joints necessary to ensure equilibrium)

# Determining DoF…

Beware ……

- Kutzbach/Grübler equation does not always works—since this equation does not consider shape or size of links. There are some exceptions ….



- Applying Kutzbach's/Gruebler's equation to this planar mechanism:

  $$m = 3(5\text{-}1)\text{-}2j_1\text{-}j_2$$

  $$m = 3(5\text{-}1)\text{-}2x6\text{-}0 = 0$$

  where, L: number of links, (=5, i.e., 4 + ground); $j_1$, number 1 DoF joints (=6), $j_2$, number of 2 or more DoF joints (=0).

- Therefore, this mechanism has: 0 DoF (should imply that motion is impossible, which we know for sure that it isn't true)

In short …

- Kutzbach/Grübler's criterion is obviously useful in determining the mobility of a wide variety of commonly used engineering mechanisms.. BUT it yields theoretical results, and can be easily misleading because it does not take geometry into account. Therefore, when an ambiguous result is obtained, the actual mobility of a mechanism must be determined by inspection.

End…

Any Questions?